IJEMD-SS, 5 (6) (2026) https://doi.org/10.54938/ijemd-ss.2026.05.6.904

International Journal of Emerging Multidisciplinaries: Social Science

*Research Paper*
*Journal Homepage: www.ojs.ijemd.com*
*ISSN (Online): 2958-0277*




# An Investigation of Foreign Exchange Market Efficiency An Analysis of Selected South Asian Economies

**1. Madiha Fayyaz, 2. Dr. Abida Yousaf**

1. PhD Scholar, International Institute of Islamic Economics (IIIE), IIU Islamabad Pakistan

madihafayyaz001@gmail.com

2. HoD, International Institute of Islamic Economics (IIIE), IIU, Islamabad, Pakistan

abida.yousaf@iiu.edu.pk

## Abstract

This paper investigates the market efficiency and scaling dynamics of foreign exchange market efficiency. The daily exchange rate return series of the four countries of South Asia, namely Bangladesh, India, Pakistan and Sri Lanka are used to test the exchange market efficiency. We used the Multifractal Detrended Fluctuation Analysis (MFDFA) to examine the non-linear dynamics of the FX market. MFDFA can investigate non-linear trends, market anomalies, and heterogeneous patterns in asset prices. Our findings suggest the departure from the traditional framework of Efficient Market Hypothesis (EMH). The empirical findings from the four markets are contrary to the weak-form random walk theory. In this regard, it indicates that the foreign exchange rate's volatility is anti-persistent in nature and hold mean reverting price dynamics. Moreover, comparative analysis shows a distinct regional hierarchy of nonlinear market complexity. Bangladesh currency market exhibits a higher degree of inefficiency and spectrum width due to the credit control and foreign exchange. The multifractal spectrum for Sri Lanka is very steep and there is a very steep singularity expansion at the end. It implies that the localized dynamics of the jumps are related to structural regime change and the sovereign debt crisis in Sri

Lanka. The results of India demonstrate superior power law and managed scaling complexity. The compressed singularity spectrum for Pakistan is a policy induced phenomenon, which has been brought about by the price control by the central bank. The overall results of this study indicate that the adaptive market hypothesis is valid. The study suggests that the discretionary interventions and liquidity constraints throw off the natural balance from the market.

**Keywords:** Foreign exchange efficiency, Multifractal detrended fluctuation analysis (MF-DFA), Generalized Hurst exponents, Singularity spectrum, South Asian economies, Adaptive market hypothesis

**JEL Classification:** C14, F31, G14, G15, O53.

# 1. Introduction

Foreign Exchange markets are a vital part of international trade, foreign investment, debt management and inflation. Control of exchange rates in emerging economies is of paramount importance in ensuring the equilibrium value of macroeconomic variables. The terms of trade, balance of payments and creditworthiness can be influenced by changes in the exchange market. Hence, understanding FX market efficiency is of great significance for investors, policymakers, financial institutions, and international businesses operating in emerging markets.

The efficient market hypothesis is regarded as a cornerstone of the asset pricing regime, meaning that the market incorporates all information quickly into market prices. Thus, prices in the market exhibit low predictive dependence. The efficient market hypothesis (EMH) was developed by Fama (1970) as a basic theory to study the behavior of financial markets. Weak form efficiency assumes that in the foreign exchange market, all the information from past market prices is fully reflected in the current exchange rate, and as a result, it does not seem possible to make money from the foreign exchange market by trying to predict returns or by trying to outsmart the market through technical trading.

Weak-form efficiency requires the return series to follow an independent and identically distributed (i.i.d.) random walk without temporal autocorrelations. The EMH suggests that exchange rate prices incorporate available information, and historical price information will not provide systematic opportunities for predicting future returns. Adeyeye et al. (2017) believe that

the absence of structural breaks and non-linearities in return series in the FX market will lead to the false perception of markets, particularly in emerging economies.

Empirical evidence suggests that the properties of foreign exchange returns are complex and violate the assumptions of the models of conventional market efficiency. Empirical evidence shows foreign exchange returns have complex statistical properties, nonlinear dependence, persistence, and volatility clustering that do not adhere to the assumptions of conventional market efficiency models. Traditional linear approaches do not fully capture these dynamics of exchange rate. FX return series have non-linear Gaussian properties (Lux & Schornstein, 2004). Standard econometric tests for weak-form efficiency (unit root tests, linear functions of autocorrelation, classical variance ratio tests, etc.) are not appropriate for higher order nonlinear dependencies. This means that an exchange rate market that is 'efficient' according to the classic linear diagnostic tests could still be highly predictable under a scale-dependent approach (Christodoulou-Volos, 2025).

The efficient market hypothesis (EMH), developed by Fama (1970), is the basic theory used to analyze the pricing of assets and exchange rate movements. When applied to FX markets, the weak-form EMH implies that all information from the market has been incorporated into the current market rates, so that it is impossible to predict any future change in the rate and that any technical trading strategy would not be profitable. Mathematically, this corresponds to the Random Walk Hypothesis (RWH), where exchange rate returns $R_T$=($P_t - P_{t-1}$) follow an independent and identically distributed (i.i.d.) stochastic process with zero mean autocorrelation:

$$\mathrm{E}[R_t \,|R_{t-1}, R_{t-2}\ldots\ldots] = \mathrm{E}[R_t]$$

Traditional econometric models for FX returns are typically based on the Gaussian distribution and linear dependence assumptions. Under a standard Brownian motion model, market returns are assumed to display monofractal scaling governed by a single Hurst exponent (H = 0.5), implying a memoryless process without long-range temporal correlations or scale-dependent fluctuations. The standard linear weak-form efficiency tests are inherently unable to detect non-linear and higher order dependence, including unit root tests, variance ratio tests and standard autocorrelation functions. However, a market labeled "efficient" based on linear diagnostic tests can still contain substantial complex, scale-dependent predictability (Jiang et al., 2019).

To address the limitations of linear finance, Peters (1994) introduced the fractal market hypothesis (FMH) by drawing on principles from econophysics and nonlinear dynamics. The FMH assumes that agents in financial markets are heterogeneous and have diverse time horizons. Participants can range from high-frequency scalpers and day traders to institutional investors and central banks (Müller et al., 1997).

In spite of that empirical evidence demonstrates the nonlinear pattern of financial markets along with long term dependence, volatility spikes and scale dependent fluctuations. Conventional tests can not capture these nonlinear trends in financial series. Multifractal methods provide a more comprehensive framework for examining the complex structure of financial time series. Multifractal Detrended Fluctuation Analysis (MFDFA) is useful in analyzing the market efficiency of financial markets. MFDFA can also identify the heterogenous dependence structure among different financial markets (Aslam et al., 2020; Raza et al., 2024; Shrestha et al., 2023). MFDFA with the estimation of Hurst exponent and multifractal spectra can identify the persistent or anti-persistent behavior of exchange rate returns. This understanding provides an approximate measure of random behavior in different fluctuations and time. The information provided by the MFDFA estimates is very beneficial for assessing the degree to which FX markets deviate from the benchmark of market efficiency (Han et al., 2020). Trade balance, remittances, inflation, and debt servicing are heavily influenced by currency market dynamics in South Asia. The management of foreign exchange in the South Asian region was previously dependent on managed floating arrangements, currency pegs, credit rationing, and capital controls. Therefore, understanding the efficiency of South Asian currency markets is important for financial regulators and investors (Noman & Ahmed, 2008).

There is growing use of multifractal techniques for empirical analysis, but there is limited evidence for South Asian countries. Our selected sample countries; Bangladesh, India, Pakistan and Srilanka display distinct macro conditions, financial market structures, exchange rate management practices, and levels of market development. These variations may lead to heterogeneous patterns of persistence, complexity, and efficiency in foreign exchange markets. However, there is a lack of comparative evidence examining FX markets of these countries within an MFDFA framework (Maganini et al., 2023).

### 1.1. Research Objectives

This study uses a multifractal detrened fluctuation analysis (MF-DFA) to address the shortcomings of linear estimation techniques and tests. Our study has the following research objectives-

- To detect and measure non-linear multifractals in exchange rate return.
- To estimate generalized Hurst exponent h(q) and to see the persistence of the process on different scales.
- To measure the multifractality in terms of the generalized Hurst spread and singularity spectrum width.
- To detect weak form market inefficiency with regard to deviations from the random walk hypothesis.
- To give an efficiency and complexity ranking to the foreign exchange markets after a comparison.
- To test the power-law scaling diagnostic parameters and structural origin of multifractality.

## 2. Literature Review

In the theory of financial economics, market efficiency is one of the core principles that assumes that the value of a financial asset is fully determined by the information of which investors are aware. According to the efficient market hypothesis (Fama, 1970), all information that can be obtained should already be reflected in the price of an asset, and no market participant should be able to achieve excess returns. The three characteristics of an efficient market are low information asymmetry, low transaction costs, and high liquidity. These properties are viewed as being necessary for efficient capital allocation and price stability. High liquidity makes trading more affordable for traders. Empirical literature indicates the presence of anomalies in financial markets. The irrational behavior of the market participants helps to explain the imperfection of the market. In the real world, these behavioral biases make market efficiency a challenge (Akin & Akın, 2024). These are the points that will be sub-optimal and should be identified and understood in the context of the market. Recent work has shown the viability of MFDFA for measuring efficiency of the foreign exchange market. In the study of Han et al. (2020) the authors analysed four of the main foreign exchange markets in the time interval between 2005 and 2019 and were able to discover strong multifractal features, with long-range correlations and fat-tailed distributions being the two

main sources of multifractality. From the analysis they also found that the market efficiency for foreign exchange market is not uniform with respect to currency and economic sub-periods.

For the foreign exchange market efficiency during the COVID-19 crisis, Aslam et al. (2020) used high-frequency data of six major currencies. The study has revealed that the markets studied are multifractal and that the pandemic has negatively affected the market efficiency, but not equally across the currencies. This suggests that foreign exchange markets could be quite sensitive to financial market turmoil and thereby significantly impact foreign exchange market dependence and efficiency.Aslam et al. (2020) examined the efficiency in the foreign exchange market during the COVID-19 crisis, by applying the high-frequency data of six major currencies. The study found evidence of multifractality in markets studied, and, at the same time, market efficiency was found to be weakened in the pandemic, but not to the same extent across the currencies. The implications of this is that foreign exchange markets are highly responsive to financial market disturbances and thus can considerably affect the dependence structure and efficiency of foreign exchange markets.

The multi fractal analysis is one of the tools presently employed to measure the efficiency in the foreign exchange market. The lower the multifractality, the more efficient the market is, and vice versa, according to MFDFA. (Datta, 2024). Market efficiency in financial markets is a dynamic phenomenon which is shaped by market participants' behaviour in different markets. Investors' behaviour is influenced by technology and regulation over different periods of time. It has moved further than the original linear random walk tests, and has advanced to more complex developments in the branch of economics known as econophysics ( Zhang & Huang, 2025). Early work using unit root tests and linear AR (autoregressive) models did not establish that the market was not efficient: linear diagnostics were not sensitive to higher-order moment dependencies.

Han et al. (2020) pointed out that there are many important multifractal properties in the major currency pairs of the global foreign exchange market of Europe, Japan and Britain. Multifractality is mainly caused in the following currency pairs by long-range correlations and fat-tailed distributions. In the case of COVID-19, Aslam et al. (2020) analysed a few high-frequency data and showed that the financial market stress increases the extent of multifractality and reduces the weak-form efficiency of the price. Unusual returns and nonlinear memory have been observed with Islamic and the fintech financial assets, which are inefficient in financial markets. Financial

markets are found to be inefficient for Islamic and fintech financial assets by employing the concept of multifractal dynamics (Shrestha et al. 2023 & Raza et al. 2024).

On both institutional and macroeconomic levels, emerging markets of South Asian countries have distinct foreign exchange markets. Using the MF-DFA method, Datta (2024) studied the Indian foreign exchange market and concluded that the Indian FX market is multifractal in nature. There is a lack of comprehensive comparative analysis for neighboring countries like Pakistan, Bangladesh and Sri Lanka. The selected economies consist of variants of the monetary policy. India has quite an established flexible inflation targeting policy by its central bank. The managed float regimes have a high level of central bank intervention with administrative credit controls and credit margin restrictions in Pakistan and Bangladesh. Sri Lanka has been a country with many monetary policy regime changes. It was found that the stock exchange markets are multifractal but there is no systematic literature review on the strength and persistence of the multifractality of the exchange rate of South Asian countries. We aim to address this research gap with this study.

# 3. Data and Methodology

This study uses the daily exchange rate series of Pakistan, Bangladesh, India, and Sri Lanka from 2000 to 2023. Daily exchange rate series of Bangladesh, India, Pakistan, and Sri Lanka are used. The daily log-returns R_t are computed as follows:

$$\boldsymbol{R_t = ln\,(P_t/P_{t-1}) \times 100}$$

$where$:

- $R_t$ FX Return on day $t$.
- $P_t$ Daily exchange rate at time $t$.
- $P_{t-1}$ Daily spot exchange rate, at previous day, at time $t-1$.

### 3.1. Comparative Analysis of Foreign Exchange Regimes

The official exchange rate regimes and key drivers of volatility in the four South Asian economies are summarized in Table 3.1.

**Table 1:** Comparative Analysis of Foreign Exchange Regimes

| Economy & Currency | Official Regime | De Facto Mechanism | Monetary Anchor & Intervention | Primary Structural Drivers |
|---|---|---|---|---|
| Bangladesh (BDT) | Managed Floating | Crawling Peg / Band Arrangement | Interest rate targeting; FX rationing during reserve stress | Garment export concentration, remittance shocks, and import L/C restrictions. |
| India (INR) | Managed Floating | Floating with Discretionary Smoothing | Flexible Inflation Targeting (FIT) via RBI; volatility smoothing | Foreign Portfolio flows (FPI), Change in dollar rate and oil shocks. |
| Pakistan (PKR) | Managed Floating | Periodic De Facto Pegs / Adjustments | Monetary aggregate targeting; SBP market interventions | Fiscal monetization, BOP crises, external debt service, and IMF targets. |
| Sri Lanka (LKR) | Flexible Exchange Rate | De Facto Defense (2020-22) to Market Float | Flexible Inflation Targeting under CBSL Act No. 16 of 2023 | Twin deficits, 2022 currency collapse, sovereign default, and MLBs. |

### 3.2. The Multifractal Detrended Fluctuation Analysis (MF-DFA) Framework

The MF-DFA algorithm established by Kantelhardt et al. (2002) was executed in five sequential steps:

Step 1: Profile Construction

Construct the cumulative mean-centered profile Y(i) of the return series R_k for i = 1, ..., N:

$$Y_i = \Sigma_{k=1}^{i}(R_k - \underline{R)}$$

$Y(i)$: Integrated cumulative profile at step $i$.

$R_k$: Logarithmic exchange rate return at time step $k$.

$\bar{R}$: Mean return over the entire time series of length $N$.

Step 2: Division into Non-Overlapping Segments

Divide profile Y(i) into N_s = int(N/s) non-overlapping segments of equal length s. To ensure that the full series is included, we repeat the process from the opposite end, yielding 2 $N_s$, segments.

Step 3: Local Trend Fitting and Variance Computation

Fit a local polynomial trend y_v(i) (using polynomial order m = 1) for each segment v = 1,2…..$N_s$, and compute the local variance F²(s, v):

$$\boldsymbol{F^2(s,v) = \frac{1}{s}\sum_{i=1}^{s} \quad \{Y[(v-1)s+i] - y_v(i)\}^2}$$

$F^2(s,v)$: Variance of the detrended fluctuation profile in segment $v$ of scale length $s$.

$s$: Scale window size (number of observations per segment).

$v$: Segment index (v = 1, 2…..$2N_s$).

$Y[\cdot]$: Integrated cumulative return profile point at index $(v-1)s+i$.

$y_v(i)$: Local polynomial trend fitted to segment $v$ (typically linear detrending, $m = 1$).

Step 4: q-Order Fluctuation Function Calculation

The average local variances across all segments to obtain the q-order fluctuation function F_q(s) are expressed as follows:

$$\boldsymbol{F_q(s) = \left\{\frac{1}{2N_s}\sum_{v=1}^{2N_s} \quad [F^2(s,v)]^{q/2}\right\}^{1/q} for\ q \neq 0}$$

For q = 0, the logarithmic averaging rule applies as follows: F_0(s) = exp{ (1 / 4N_s) ∑_{v=1}^{2N_s} ln[ F²(s, v) ] }.

$F_q(s)$: $q$-th order overall fluctuation function evaluated at window scale length $s$.

$q$: Fluctuation moment order index ($q \in R$, typically evaluated over $[-5,+5]$).

- Negative moments ($q < 0$) indicate small-scale fluctuations.
- Positive moments ($q > 0$) indicate large-scale fluctuations.

$N_s$: Number of non-overlapping segments of scale size $s$

$F^2(s,v)$: The local detrended variance of segment $v$

Step 5: Power-Law Scaling and Generalized Hurst Exponents

For the range of scales, investigate the power law scaling behavior by regressing ln F_q(s) against ln s.For the range of scales s, examine the power law scaling behavior by regressing ln F_q(s) against ln s.

$$\boldsymbol{ln\, F_q(s) = h(q) ln\, s + C}$$

### 3.3. Derivation of Mass Exponents t(q) and Singularity Spectrum f(α)

The mass exponent t(q) (or Rényi exponent τ(q)) is derived directly from h(q) as follows:

$$t(q) = q * h(q) - 1$$

Applying the Legendre transform converts t(q) into the singularity strength α and the multifractal spectrum f(α):

$$\alpha = \boldsymbol{h(q) + q \cdot h'(q)}$$

$$\boldsymbol{f(\alpha) = q[\alpha - h(q)] + 1}$$

$\alpha$ **(Hölder Singularity Exponent)**: Measures the localized singularity strength (intensity of price fluctuations) at a specific time point. Lower or negative values indicate sharp, non-differentiable price jumps and extreme volatility.

$h'(q)$: First derivative of the generalized Hurst exponent with respect to moment order $q$ ($\frac{dh(q)}{dq}$).

$f(\alpha)$: Multifractal singularity spectrum, representing the Hausdorff fractal dimension of the subset of data points characterized by singularity exponent $\alpha$.

$q$: Statistical fluctuation order index ($q \in [-5, +5]$

The key summary metrics of multifractality are:

1. **Generalized Hurst Spread:** $\Delta h = h(-5) - h(+5)$
2. **Singularity Spectrum Width:** $\Delta\alpha = \alpha_{max} - \alpha_{min}$

# 4. Results and Discussion

### 4.1. Summary of Descriptive Statistics

The descriptive statistics of the daily log returns of exchange rates for Bangladesh, India, Pakistan and Sri Lanka are presented in Table 4.1. The average daily returns are comparatively small for all four markets as they lie within the range of 0.013 in India to 0.033 in Pakistan. The standard deviation, on the other hand, had much greater variation. The Bangladesh standard deviation is 1.399 whereas India is 2.228, Pakistan is 2.592 and Sri Lanka is 3.665. The statistics show significant variations in terms of spread of daily exchange rate returns across the four markets. In addition, the range of minimums and maximums shows that there was considerable fluctuation in exchange rates. The largest observed range is seen in Sri Lanka between −60.216 and 56.314. The

range is quite narrow in Bangladesh, from −20.003 to 9.912. The variations indicate that the markets selected do not have uniform volatility conditions for returns.

**Table 2:** Descriptive Statistics

| Country | Observations | Mean Return | Std. Deviation | Minimum | Maximum |
|---|---|---|---|---|---|
| Bangladesh | 5,778 | 0.018 | 1.399 | -20.003 | 9.912 |
| India | 5,887 | 0.013 | 2.228 | -24.531 | 13.189 |
| Pakistan | 6,748 | 0.033 | 2.592 | -32.017 | 23.679 |
| Sri Lanka | 6,141 | 0.020 | 3.665 | -60.216 | 56.314 |

### 4.2. MF-DFA Empirical Scaling Results

Table 3 shows that the generalized Hurst exponents h(q) at q = {-5, 2, +5} obtained using MF-DFA, Hurst spread Δh, singularity spectrum parameters (α_min, α_max, Δα), mean $R^2$, and overall market efficiency ranks.

**Table 3:** MFDFA Scaling Results

| Country | h(-5) | h(2) | h(+5) | Δh | α_min | α_max | Δα | Mean $R^2$ | Efficiency Rank |
|---|---|---|---|---|---|---|---|---|---|
| Bangladesh | 1.386 | 0.301 | 0.059 | 1.327 | -0.141 | 1.626 | 1.766 | 0.846 | Rank 4 (Least Efficient) |
| India | 0.962 | 0.218 | 0.083 | 0.879 | -0.030 | 1.175 | 1.205 | 0.951 | Rank 2 |
| Pakistan | 0.882 | 0.086 | 0.009 | 0.872 | -0.045 | 0.996 | 1.041 | 0.703 | Rank 1 (Narrow Spectrum) |
| Sri Lanka | 0.904 | 0.156 | 0.064 | 0.840 | -0.013 | 1.201 | 1.214 | 0.790 | Rank 3 |

Empirical results derived using Multifractal Detrended Fluctuation analysis (MF-DFA) are used to establish the nonlinear multifractal dynamics in all four foreign exchange return series of South Asian economies for daily observations. The Hurst exponent, h(q), as shown in Table 4.2 is not fixed. It has a descending trend as q increases from (-5) to (+5) over the entire region of Bangladesh, India, Pakistan and Sri Lanka. This nonlinear scaling behaviour is in direct contrast to the classical linear financial models which assume monofractality of foreign exchange returns in South Asia, thus proving that the memory structures of foreign exchange returns are scale dependent. The generalized Hurst exponents indicate that there is marked scale dependence in the relationship between small and large scale return fluctuations.

In case of negative moments (q<0), the value of h (-5) ranges from 0.882 in Pakistan to highest value being in Bangladesh i.e. 1.386. All the values of h(-5) are much greater than 0.5 and the values have a high degree of positive autocorrelation at long lags. In the case of small shocks, they tend to get larger in the same direction, suggesting herding and delays in the information processing of market participants at the local level. The extremely high value of h(-5) >1 in Bangladesh shows that the noise dynamics are not stationary with trend-reinforcing properties at small fluctuation scales.

At standard Detrended Fluctuation Analysis $q = 2$, value of $h(2)$ values range from 0.086 to 0.301. The lowest value is for Pakistan and the highest value is attained by Bangladesh. All $h(2)$ values deviate substantially below the weak-form Efficient Market Hypothesis (EMH) benchmark, which is $h(2) < 0.5$. If value of $h(2)$ is exactly equal to 0.5, then that market would be considered to follow random walk. But our results suggests strong anti persistence $h(2) < 0.5$ proves that standard daily return volatility is characterized by frequent mean-reverting adjustments. Large scale fluctuations capture at the value of q=+5. For positive moments ($q > 0$) and $h(+5)$ approaches zero, from 0.009 to 0.083, across all four currencies. This pronounced anti persistence

At standard Detrended Fluctuation Analysis $q = 2$, value of $h(2)$ values range from $0.086$ to $0.301$. The lowest value is for Pakistan and the highest value is attained by Bangladesh. All $h(2)$ values deviate substantially below the weak-form Efficient Market Hypothesis (EMH) benchmark, which is $h(2) < 0.5$. If value of $h(2)$ is exactly equal to 0.5, then that market would be considered

to follow random walk. But our results suggests strong anti persistence $h(2) < 0.5$ proves that standard daily return volatility is characterized by frequent mean-reverting adjustments.

Large scale fluctuations capture at the value of q=+5. For positive moments ($q > 0$) and $h(+5)$ approaches zero, from $0.009$ to $0.083$, across all four currencies. This pronounced anti persistence indicates that extreme price shocks, large depreciations or appreciations undergo rapid, aggressive mean reversion. This can prevent long-term trends in major fluctuations due to direct central bank smoothing interventions or capital controls.

In econophysics literature, the generalized Hurst exponent spread is calculated with $\Delta h = h(q_{min}) - h(q_{max})$. The multifractal spectrum width i.e. $\Delta\alpha = \alpha_{max} - \alpha_{min}$ can be used as direct quantitative metrics of market inefficiency. A wider spectrum ($\Delta\alpha$) and a larger variation ($\Delta h$) reflect higher structural complexity. Large values $\Delta\alpha$ and $\Delta h$ depicts stronger nonlinear dependencies and deviation from market efficiency. In our results, Bangladesh has $\Delta h = 1.327$ and $\Delta\alpha = 1.766$, which makes it the least efficient economy in our sample. This severe market inefficiency could be explained by existence of strict credit control, rationing of foreign currency and export conversion mandates.The of $\alpha_{min}$ is $-0.141$ this negative sign highlights the riskiness of Bangladesh's currency returns.

The results from India and Sri Lanka show intermediate complexity. Value of $\Delta h$ for India is $0.879$ and for Sri Lanka is$0.840$. The value of $\Delta\alpha$ for India is $1.205$ and for Srilanka is $1.214$. These findings show moderate multifractal strength in both countries. The Indian FX market operates under a managed float. Periodic intervention by the Reserve Bank of India (RBI) during global shocks introduces nonlinear scaling dependencies. Sri Lanka's findings reflect extreme shifts in the macroeconomic regime and currency devaluation shock of 2022.

Our results of the multifractal spectrum and generalized Hurst spread make Pakistan a relatively efficient economy among our sample countries. But low value of $h(2)$ and $h(5)$ signal that this lower spectrum width is not due to random-walk. It might be the result of intense currency management practices of the central bank.

### 4.3. Power-Law Scaling Diagnostics and Goodness-of-Fit Analysis

Table 4 lists the scaling range, number of logarithmic scale windows, minimum, maximum, and mean $R^2$ and scaling quality assessments.

**Table 4:** Power-Law Scaling Diagnostics and Goodness-of-Fit Analysis

| Country | Scale Range (s) | No. Scales | Min $R^2$ | Max $R^2$ | Mean $R^2$ | Scaling Quality Assessment |
|---|---|---|---|---|---|---|
| India | 60-800 | 17 | 0.767 | 0.997 | 0.951 | Moderate-Strong Power-Law Fit |
| Bangladesh | 90-702 | 13 | 0.259 | 0.978 | 0.846 | Weak / Review at Extreme q |
| Sri Lanka | 80-800 | 14 | 0.316 | 0.973 | 0.790 | Weak / Review (Intervention Distortions) |
| Pakistan | 80-800 | 14 | 0.099 | 0.885 | 0.703 | Weak / Review (Heavy Pegging Distortions) |

Table 4 presents a comparative goodness-of-fit analysis. $MeanR^2$ for India is $0.951$ with range of $0.767 \leq R^2 \leq 0.997$. This demonstrates superior power-law scaling fidelity across all fluctuation moments. This represents the continuous, robust power-law scaling supported by liquid FX markets managed through flexible inflation targeting (FIT). These findings are consistent with the implications of the Hurst exponent.

For Bangladesh $MeanR^2$ is $0.846$ and it exhibits strong overall linear fitting. At extreme negative moments $(q = -5)$ $R^2$ drops to $0.259$ . This lower value of $R^2$ indicates localized scaling

curvature during small-scale fluctuation. These fluctuations may be caused by administrative FX rationing and trade credit restrictions. coefficients of determination for Pakistan and Sri Lanka is 0.703 and 0.790 respectively. $MinR^2$ of Pakistan falls to 0.099 and Sri Lanka's reaches a lower level of 0.316. This might be due to heavy central bank intervention in Pakistan. In 2022 Sri Lanka's structural currency collapse is the reason to distort the fluctuation curves.

### 4.4. Scale-Dependent Memory and Weak-Form EMH Rejection

Table 5 isolates the standard Hurst exponents h(2) and provides behavioral interpretations relative to the random-walk benchmark (h = 0.500).

**Table 5:** Standard Hurst Exponents h(2)

| Country / Currency | Standard Hurst h(2) | Scaling Interpretation | Underlying Market Dynamic |
|---|---|---|---|
| Bangladesh (BDT) | 0.301 | Anti-persistent | Moderate mean reversion; frequent administrative adjustments and foreign exchange (FX) rationing. |
| India (INR) | 0.218 | Anti-persistent | Discretionary volatility smoothing by the RBI within a flexibly managed float. |
| Sri Lanka (LKR) | 0.156 | Anti-persistent | Strong mean-reverting corrections triggered by reserve defenses and devaluation shocks. |
| Pakistan (PKR) | 0.086 | Anti-persistent | Extreme mean reversion is driven by heavy central bank currency management and price caps. |

As established in Table 5, the estimated standard Hurst exponents $h(2)$ across all four South Asian economies fall significantly below the theoretical benchmark of 0.500. It ranges from 0.301 for Bangladesh down to 0.086 for Pakistan. This universal deviation calculated by h(2) - 0.500 < 0

provides conclusive empirical evidence to reject the weak-form Efficient Market Hypothesis across South Asian foreign exchange markets. Daily exchange rate returns do not follow a Gaussian random walk. However, price movements show systematic and non-random dynamic structures. The range of generalized hurst exponent $0 < h(2) < 0.500$ characterizes an anti-persistent or mean-reverting time series. Anti persistence implies that an upward price movement on day $t$ is statistically more likely to be followed by a downward adjustment on day $t + 1$, and vice versa. The degree of anti persistence intensifies as $h(2)$ approaches zero.

For Bangladesh h (2) is 0.301 and Deviation is 0.199. It exhibits the highest h(2) value out of all the regions in the sample, and so has moderate anti persistence. The daily returns show mean reverting tendencies while the value of h(2) is close to 0.3 which is associated with market friction. Foreign currency rationing on administration, margin requirements for import, and currency conversion requirements are present in Bangladesh. Such market pressures can retard the adjustment of prices to the short run equilibrium. So this gradual adjustment process will keep some noise at small scales and impose mean reversion at larger scales.

For India the value of h(2) is 0.218 and Deviation is 0.282. There is a definite anti persistence in the Indian economy as reflected in these values. The Indian currency market has a very well managed floating rate regime. The Reserve Bank of India (RBI) routinely carries out volatility smoothing. When there are any excessive exchange rate fluctuations, RBI intervenes in the buying and selling of foreign exchange reserves. This smoothing mechanism is active, meaning that exchange rate returns move towards some central tendency on a systematic basis, creating predictable mean-reverting cycles. Value of h(2) is 0.156 and deviation from benchmark level is 0.344 for SriLanka. That's a good sign of anti-persistent scaling. In 2020-2022, the central bank intervened in the market to support the rigid exchange rates, resulting in high price suppression. International reserves were exhausted, then forced currency floatations and devaluation shocks occurred, bringing about sudden reversals. This resulted in a very anti-persistent return.

Results for Pakistan are h(2)=0.086 and Deviation is 0.414. Lowest value of h(2) in the region indicates extreme anti persistence. When an exponent gets closer to zero, it clearly shows that there are heavy stabilization processes going on in the standard price movements of the Pakistani Rupee on the daily basis. Daily price fluctuations in the market are kept subdued with the help of policy

measures by the State Bank of Pakistan (SBP), administrative price caps and crawling peg. It enforces mean reverting at an aggressive rate to any transient deviation from the policy. All economies showed evidence of strong anti persistence h(2)<0.5 and this was confirmed in our results. This suggests that technical trading rules using mean-reversion indicators will be able to take advantage of nonlinear predictability and earn risk-return premiums in the South Asian foreign exchange markets.

### 4.5 Comparative Inefficiency Analysis (Δh vs. Δα)

Table 5 compares the hurst spread (Δh) ranks against the local singularity spectrum width (Δα) ranks. (Δa) highlighting the key structural drivers of ranking shifts. To evaluate the degree of nonlinear complexity and market inefficiency across South Asian foreign exchange markets, two complementary multifractal metrics are employed: the generalized Hurst exponent spread and the multifractal spectrum width.

Bangladesh holds Rank 1 across both metrics ($\Delta h = 1.327,\ \Delta\alpha = 1.766$) and highest market inefficiency. Results for Bangladeshi shows that its returns are global scale-dependent memory, which is shown by the steep h(q) slope. The wide range of singularity parabola represents extreme localized jump Severe liquidity bottlenecks exist due to administrative foreign currency rationing, import margin requirements, and export proceed conversion mandates. These restrictions cause the noise to become nonstationary at small scales h(-5)=1.386 and increase nonlinear complexity.

**Table 6:** Comparative Dual-Metric Inefficiency Rankings Across Selected Foreign Exchange Markets

| Country / Currency | Hurst Spread (Δh) | Δh Rank | Spectrum Width (Δα) | Δα Rank |
|---|---|---|---|---|
| Bangladesh (BDT) | 1.327 | Rank 1 | 1.766 | Rank 1 (Least Efficient) |
| India (INR) | 0.879 | Rank 2 | 1.205 | Rank 3 |
| Pakistan (PKR) | 0.872 | Rank 3 | 1.041 | Rank 4 (Narrow Spectrum) |

**Table 6:** Comparative Dual-Metric Inefficiency Rankings Across Selected Foreign Exchange Markets

| Country / Currency | Hurst Spread (Δh) | Δh Rank | Spectrum Width (Δα) | Δα Rank |
|---|---|---|---|---|
| Sri Lanka (LKR) | 0.840 | Rank 4 | 1.214 | Rank 2 (Tail-Shock Effect) |

Sri Lanka displays a lower generalized Hurst spread and Rank 4. But it jumps to Rank 2 in spectrum width ($\Delta\alpha = 1.214$). While Sri Lanka's generalized Hurst exponent curve shifts relatively smoothly across intermediate moments. In 2022, currency collapsed, where the Rupee depreciated by over 80% following international reserve exhaustion and sovereign default. It introduced extreme localized price singularities. Because $\Delta\alpha$ heavily penalizes extreme fat-tailed jump events, the 2022 shock dramatically expands the singularity spectrum ($\Delta\alpha$) despite a narrower smooth spread ($\Delta h$).

India ranks 2 in $\Delta h$ 0.879 and 3 in $\Delta\alpha$ (1.205). Operating under a deep interbank market with Flexible Inflation Targeting (FIT), the Reserve Bank of India's (RBI) active volatility smoothing introduces moderate scale-dependent memory across different time horizons ($\Delta h$). However, high market liquidity and flexible exchange rate adjustments prevent unanchored price jumps, keeping its spectrum width ($\Delta\alpha$) lower than those of Bangladesh and Sri Lanka. Pakistan displays the narrowest singularity spectrum width ($\Delta\alpha = 1.041$, Rank 4). In line with the scaling diagnostic findings (Mean $R^2 = 0.703$), this narrow spectrum is not an indicator of random-walk efficiency. Persistent State Bank of Pakistan(SBP) interventions suppress large-scale price movements. Central bank interventions like administrative price caps and crawling peg artificially compressing the singularity spectrum capping $\alpha_{max} = 0.996$.

### 4.6. Mass Exponent Spectrum t(q) and Nonlinear Concavity Analysis

Table 4.6 lists the value of mass exponents across different statistical moments $q$. These values demonstrate a downward nonlinear pattern of mass exponent spectrum.

**Table 7:** Mass Exponent Spectrum $t(q)$ and Nonlinear Concavity Analysis Across Selected Foreign Exchange Markets

| Country / Currency | t(-5) | t(0) | t(2) | t(+5) |
|---|---|---|---|---|
| Bangladesh (BDT) | -7.930 | -1.000 | -0.398 | -0.705 |
| India (INR) | -5.810 | -1.000 | -0.564 | -0.585 |
| Sri Lanka (LKR) | -5.520 | -1.000 | -0.688 | -0.680 |
| Pakistan (PKR) | -5.410 | -1.000 | -0.828 | -0.955 |

The relation of Mass exponent to generalized hurst exponent is explained in following relation:

$$t(q) = \tau(q) = q \cdot h(q) - 1$$

As demonstrated in Table 4.6, $t(0) = -1.000$ holds identically across all four South Asian economies, satisfying the exact mathematical boundary condition where $t(0) = 0 \cdot h(0) - 1 = -1.000$.

### *4.6.1. Rejection of Monofractal Scaling and Empirical Concavity*

The empirical values of $t(q)$ display severe nonlinear departures from a straight line across all four South Asian currency markets. Bangladesh has the steepest nonlinear curve in the region. It drops from $t(-5) = -7.930$ to $t(2) = -0.398$ before turning downward to $t(+5) = -0.705$. Our results imply that small scale fluctuations possess. The extreme divergence confirms that small-scale fluctuations possess high scale hurst exponent. It reflects the existence of persistent noise driven by administrative credit rationing and foreign currency allocation constraints.

India displays a smooth, nonlinear progression from $t(-5) = -5.810$ to $t(+5) = -0.585$. The change in slope ($\frac{dt(q)}{dq} = h(q) + q \cdot h'(q) = \alpha$) across moment orders confirms that the Indian Rupee is characterized by an irregular behavior. For Sri Lanka $t(-5) = -5.520$ and $t(+5) = -0.680$. Results of Pakistan are $t(-5) = -5.410$ and $t(+5) = -0.955$. For Pakistan, the value

for $t(+5)$ approaches near $-1.000$, which reflects that central bank price management with fixing the cap level of exchange rate suppresses the large fluctuation in the market.

The nonlinear mass exponent function $t(q)$ forms the direct mathematical input for generating the multifractal singularity spectrum $f(\alpha)$ via the Legendre transform:

$$\alpha = \frac{dt(q)}{dq} = h(q) + q \cdot h'(q)$$

$$f(\alpha) = q \cdot \alpha - t(q) = q[\alpha - h(q)] + 1$$

The pronounced downward concavity of $t(q)$ across all four South Asian economies guarantees that their corresponding singularity spectra $f(\alpha)$ form single-humped, inverted parabolas ($\Delta\alpha > 0$). Our results support the Adaptive Market Hypothesis (AMH) and show the level of market efficiency across different markets.

### 4.7. Graphical Analysis and Visual Inspection

Figures 1–4 illustrate the extracted scaling functions, singularity spectra, comparative inefficiency rankings, and mass exponent concavity curves, which visually validate the empirical findings.

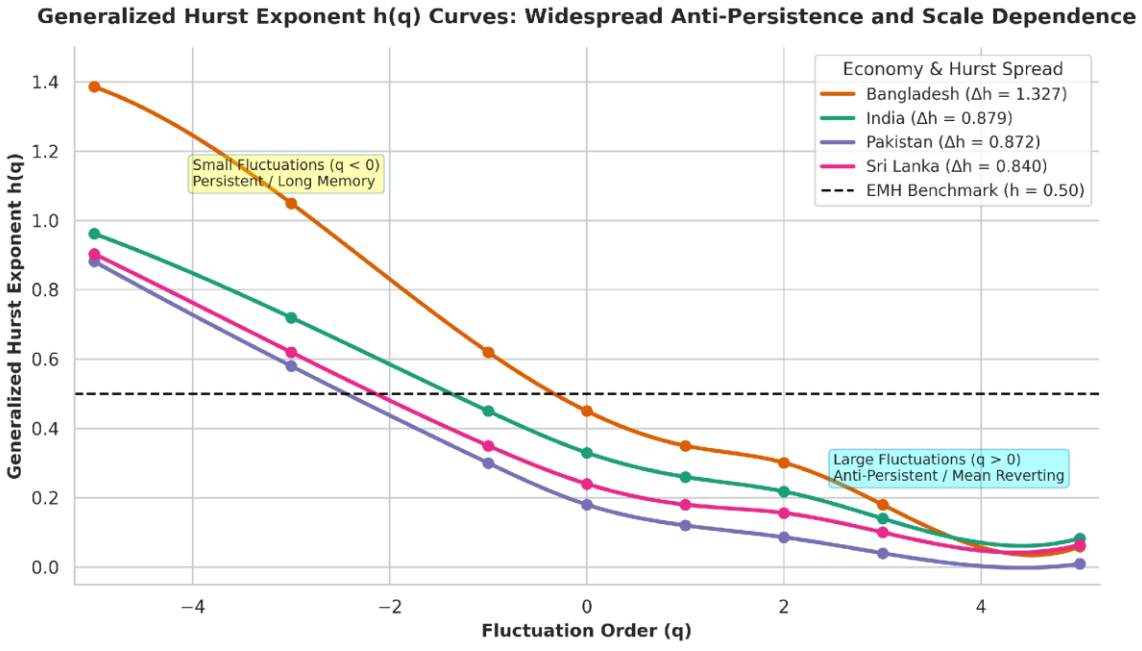



*Figure 1: h(q) vs. q*

Figure 1 illustrates the consistent decrease of the generalized Hurst exponent h(q) as the value of q increases from -5 to +5 across all four currencies. For the lower value of moment order ($q < 0$), curves for all countries lie well above the EMH baseline i.e. $h = 0.500$. For Bangladesh value is highest at $h(-5) = 1.386$ and for India it is at 0.962 at the same level. It shows persistent memory patterns in daily returns fluctuations. At $q = 2$, all four curves drop distinctly below 0.500 ($0.086 \leq h(2) \leq 0.301$), it shows mean reverting behavior of data. For positive moments ($q = +5$), h(q) approaches zero ($0.009 \leq h(+5) \leq 0.083$), reflecting policy-enforced anti persistence.

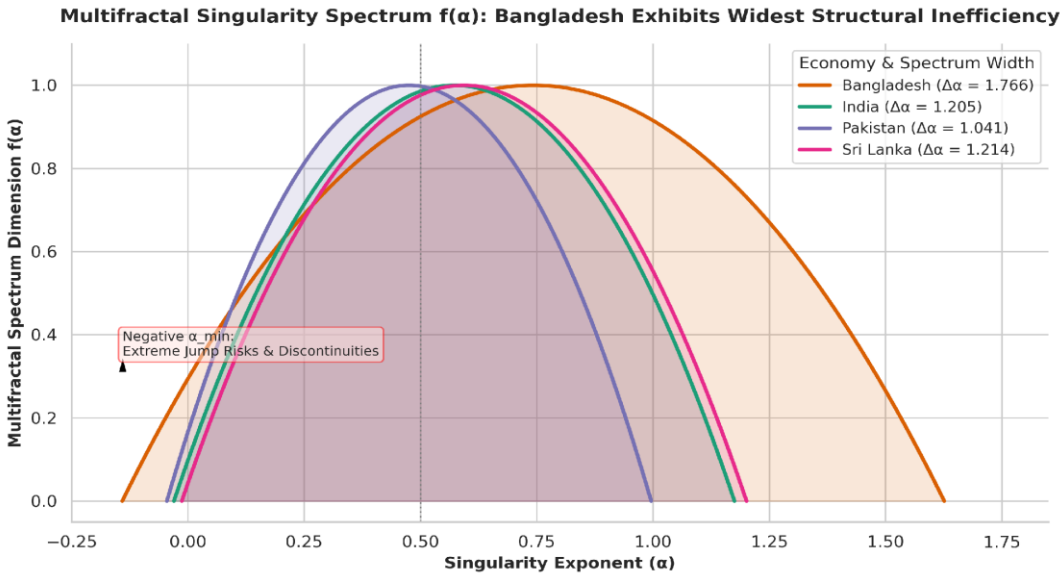



*Figure 2: f(α) vs. α*

Figure 2 illustrates the inverted parabolic singularity spectra, f(α), obtained through Legendre transformation. Bangladesh presents the widest spectrum, with Δα measuring 1.766, covering the range $\alpha \in [-0.141, 1.626]$. This indicates the highest structural complexity and nonlinear inefficiency. Sri Lanka has a value of $\Delta\alpha = 1.214$ and India's $\Delta\alpha = 1.205$ show a moderate spectrum width. Pakistan, with $\Delta\alpha = 1.041$, has a narrower and more compressed spectrum, attributed to significant currency defenses by the State Bank of Pakistan (SBP). Notably, all four spectra reach negative $\alpha_{min}$ values: -0.141 for BDT, -0.045 for PKR, -0.030 for INR, and -0.013 for LKR. This demonstrates the existence of indistinguishable price singularities and significant tail-risk jump dynamics.

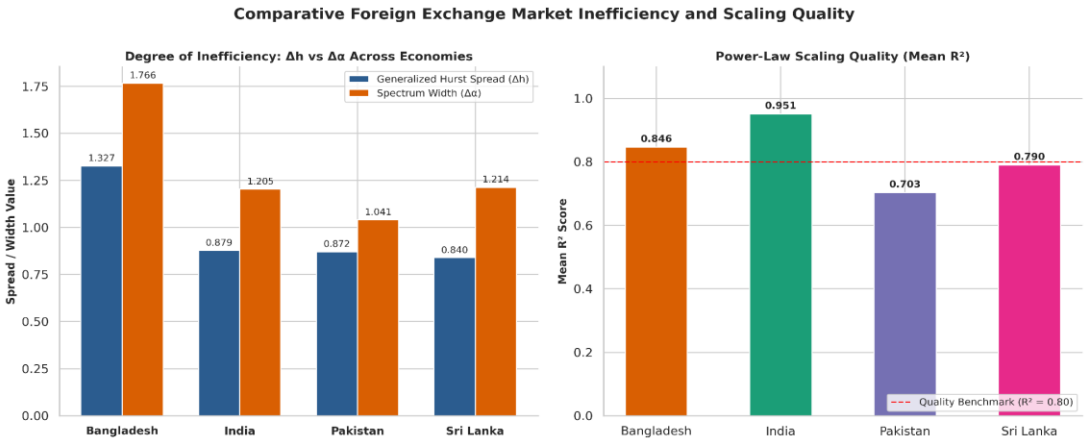



*Figure 3: Comparative Rankings of Δh vs. Δα and Mean R².*

Figure 3 provides a comparative analysis through a dual-panel format. The left panel illustrates that Bangladesh ranks highest in both inefficiency metrics. In contrast, Sri Lanka exhibits a high spectrum width $\Delta\alpha = 1.214$ compared to its Hurst spread $\Delta h = 0.840$, attributed to heavy-tailed price singularities following its 2022 currency collapse. The right panel indicates that India demonstrates superior fidelity in power-law scaling, with a Mean $R^2$ of 0.951. Conversely, Pakistan (Mean $R^2 = 0.703$) and Sri Lanka (Mean $R^2 = 0.790$) experience scaling distortions due to central bank price interventions and significant regime disruptions.

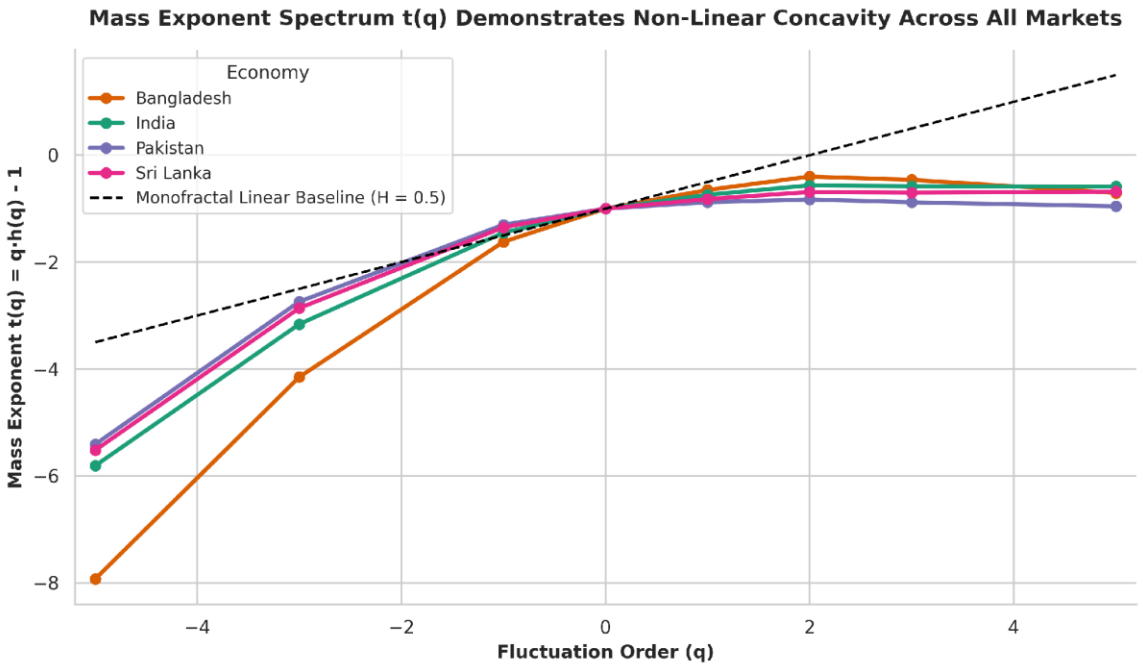



*Figure 4: Mass exponent spectrum t(q) = q*h(q) - 1*

Figure 4 illustrates the mass exponent spectrum, denoted as $t(q) = q*h(q) - 1$. According to the classical monofractal hypothesis, where $h(q) = H = 0.500$, $t(q)$ is represented by a straight dashed line, $t(q) = 0.5q - 1$. However, the empirical curves for all four South Asian economies demonstrate significant nonlinear downward concavity. This observation provides definitive mathematical evidence of multi-scaling behavior, thereby refuting simple monofractal models.

Examining the empirical findings from South Asia reveals important macroeconomic insights. Bangladesh, with a $\Delta h$ of 1.327 and $\Delta\alpha$ of 1.766, ranks fourth in terms of extreme multifractality. This is attributed to stringent foreign exchange controls, obligatory conversion of export revenues, and administrative constraints on import letters of credit. Sri Lanka, with a singularity spectrum width of $\Delta\alpha = 1.214$, holds the third position, reflecting the severe tail risk caused by the 2022 balance of payment crisis and sovereign default. India, ranked second, exhibits managed complexity with $\Delta h = 0.879$ and $\Delta\alpha = 1.205$, alongside an impressive power-law scaling indicated by a Mean $R^2$ of 0.951. This suggests that its flexible inflation targeting framework supports natural price discovery while reducing erratic fluctuations. Lastly, Pakistan's compressed spectrum, ranked first with $\Delta\alpha = 1.041$ and a low Mean $R^2$ of 0.703, indicates artificial suppression by the State Bank of Pakistan's crawling peg defenses rather than true random-walk efficiency.

# 5. Conclusion and Policy Implication

The aim of this study is to analyze the potential effectiveness and the nonlinear complexity of foreign exchange markets. Daily currency returns of major South Asian countries; Bangladesh, India, Pakistan and Sri Lanka are analyzed. In this study, multifractal detrended fluctuation analysis (MF-DFA) was used to tackle six main objectives in a systematic manner. The Hurst exponent, h(q) was always monotonously decreasing for all values of q in the range -5 to +5. All four markets had been found to be universally multi-fractal with the mass exponent t(q) shown to be concave (nonlinear). The results for small fluctuations (q = -5) exhibited very strong persistence, whereas the results for standard volatility (q = 2) exhibited anti persistence throughout. If there is a large fluctuation (q = +5), then it implies that there is a rapid forced mean reversion. Our results suggest that the least efficient is Bangladesh, followed by Sri Lanka, India and Pakistan.

Across South Asia, the empirical deviation of h(2) from 0.500 is universally counter to the weak form efficient market hypothesis (EMH) and the random walk hypothesis. Long range temporal correlations lead to multifractality. The results are important for the fields of nonlinear asset pricing and international finance. Scale dependent dependence in the returns of emerging currencies is not described sufficiently by linear Gaussian asset pricing models and the standard variance ratio test. FX price efficiency is a developing feature shaped by institutional development, exchange rate regime flexibility and policy intervention intensity. Rigid price bands by the central bank are interfering with price changes. These efforts result in long memory dependence and in higher order of nonlinear complexity. Based on our results, the administration price ceilings, margin requirements and unanchored reserve requirements should be gradually narrowed in Pakistan and Bangladesh. The central bankers should eliminate direct central bank buying of government bills of credit. This will stop monetary financing from depleting foreign reserves and leading to pass-through inflation. This work can be expanded in the following three ways in future: First, it can provide dynamic Hurst exponents that can be used to monitor time-varying efficiency for certain crisis subperiods. Another possible research direction is to use asymmetric MF-DFA to detect the difference in the scaling characteristics of bull and bear markets. Third possibility is to use data at higher frequency to study the market microstructure and liquidity dynamics.

## References

[1] Aslam, F., Aziz, S., Nguyen, D. K., Mughal, K. S., & Khan, M. (2020). On the efficiency of foreign exchange markets in times of the COVID-19 pandemic. Technological Forecasting and Social Change, 161, Article 120261. https://doi.org/10.1016/j.techfore.2020.120261

[2] Akin, I., & Akin, M. (2026). Behavioral finance impacts on US stock market volatility: An analysis of market anomalies. Behavioural Public Policy, 10(2), 337–361.

[3] Bertelsmann Stiftung. (2024). BTI 2024 country report: Sri Lanka. Bertelsmann Stiftung.

[4] Central Bank of Sri Lanka [CBSL]. (2026). Market operations report – June 2026. Central Bank of Sri Lanka.

[5] Christodoulou-Volos, C. (2025). The impact of temporal aggregation on nonlinearity detection: An empirical analysis of exchange rate dynamics using the LM test. Edelweiss Applied Science and Technology, 9(7), 918–936.

[6] Datta, R. P. (2024). Analysis of Indian foreign exchange markets: A multifractal detrended fluctuation analysis (MFDFA) approach. International Journal of Empirical Economics, 3(3). https://doi.org/10.1142/S2810943024500069

[7] Diniz-Maganini, N., Rasheed, A. A., & Sheng, H. H. (2021). Exchange rate regimes and price efficiency. Journal of International Financial Markets, Institutions and Money, 73, Article 101361. https://doi.org/10.1016/j.intfin.2021.101361

[8] Diniz-Maganini, N., Rasheed, A. A., & Sheng, H. H. (2023). Price efficiency of the foreign exchange rates of BRICS countries. Latin American Journal of Central Banking, 4(4), Article 100081. https://doi.org/10.1016/j.latcb.2023.100081

[9] Dissanayaka, U. (2024). Twin deficit hypothesis: The case of Sri Lanka (Master's report, University of New Brunswick). UNB Scholar.

[10] Dutta, S., Ghosh, D., & Chatterjee, S. (2016). Multifractal detrended cross-correlation analysis of foreign exchange and SENSEX fluctuation in Indian perspective. Physica A: Statistical Mechanics and its Applications, 463, 188–201. https://doi.org/10.1016/j.physa.2016.07.027

[11] Fama, E. F. (1970). Efficient capital markets: A review of theory and empirical work. The Journal of Finance, 25(2), 383–417. https://doi.org/10.2307/2325486

[12] Han, C., Wang, Y., & Xu, Y. (2020). Nonlinearity and efficiency dynamics of foreign exchange markets: Evidence from multifractality and volatility of major exchange rates. Economic Research-Ekonomska Istraživanja, 33(1), 731–751. https://doi.org/10.1080/1331677X.2020.1734852

[13] Jabeen, M., & Ahmed, U. (2015). Exchange rate volatility and market efficiency: Evidence from Pakistan. AIUB Journal of Business and Economics, 12(1), 70–92.

[14] Jayasekara, M. (2022). An analysis of exchange rate pass-through and impact on domestic inflation in Sri Lanka. Indian Journal of Economics and Financial Issues, 3(1), 1–31.

[15] Kantelhardt, J. W., Zschiegner, S. A., Koscielny-Bunde, E., Havlin, S., Bunde, A., & Stanley, H. E. (2002). Multifractal detrended fluctuation analysis of nonstationary time series. Physica A: Statistical Mechanics and its Applications, 316(1–4), 87–114. https://doi.org/10.1016/S0378-4371(02)01383-8

[16] Makovský, P. (2021). Is the Market Efficiency Hypothesis applicable in the Czech Republic – FOREX case. Universal Journal of Accounting and Finance, 9(6), 1495–1502. https://doi.org/10.13189/ujaf.2021.090626

[17] Mandelbrot, B. (1963). The variation of certain speculative prices. The Journal of Business, 36(4), 394–419.

[18] Müller, U. A., Dacorogna, M. M., Davé, R. D., Olsen, R. B., Pictet, O. V., & von Weizsäcker, J. E. (1997). Volatility of foreign exchange markets: A study of magnifying effects. Journal of Empirical Finance, 4(2–3), 213–239. https://doi.org/10.1016/S0927-5398(97)00008-0

[19] Olufemi, A. P., Adewale, A. O., & Oseko, M. S. (2017). Efficiency of foreign exchange markets in sub-Saharan Africa in the presence of structural break: A linear and non-linear testing approach. Journal of Economics and Behavioral Studies, 9(4), 122–131.

[20] Peters, E. E. (1994). Fractal market analysis: Applying chaos theory to investment and economics. John Wiley & Sons.

[21] Rathnayake, G. I. (2018). Effect of exchange rate volatility on Sri Lanka's inbound tourist flow. Central Bank of Sri Lanka Staff Studies, 48(2), 25–68.

[22] Raza, S. A., Shah, N., & Suleman, M. T. (2024). A multifractal detrended fluctuation analysis of Islamic and conventional financial markets efficiency during the COVID-19 pandemic. International Economics, 177, Article 100463. https://doi.org/10.1016/j.inteco.2023.100463

[23] Rothschild & Co. (2024). Important innovations in sovereign finance: The arrival of Macro-Linked Bonds (MLBs) and Governance-Linked Bonds (GLBs). Rothschild & Co Insights.

[24] Shrestha, K., Naysary, B., & Philip, S. S. S. (2023). Fintech market efficiency: A multifractal detrended fluctuation analysis. Finance Research Letters, 54, Article 103775. https://doi.org/10.1016/j.frl.2023.103775

[25] Vigato, A. (2025). Sri Lanka: After the storm, a promising economic recovery (MacroDev No. 128). Éditions AFD.

[26] World Bank. (2024). Sri Lanka Development Update: Opening Up to the Future. World Bank Group.

[27] Zhang, W., & Huang, Z. (2025). Nonlinear dynamics of RMB exchange rate volatility: A multifractal perspective within the G-expectation framework. Fractal and Fractional, 9(11), 746.