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Neutrosophic Delta-Beta Connected Topological Spaces

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Abstract

The real-life situations always include indeterminacy. The Mathematical tool which is well known in deal with indeterminacy is neutrosophic. The notion of neutrosophic set is generally referred to as the generalization of intuitionistic fuzzy set. In this paper, the notion of neutrosophic $\delta\beta$ -connectedness and $\delta\beta$ -disconnectedness in neutrosophic topological spaces is introduced. Also, we introduce neutrosophic $\delta\beta$ -separated sets, neutrosophic super $\delta\beta$ -connected spaces, neutrosophic extremely $\delta\beta$ -disconnected spaces and neutrosophic strongly $\delta\beta$ -connected spaces. We investigate and study several properties and characterizations concerning connectedness in these spaces.

Keywords: neutrosophic topology, neutrosophic $\delta\beta$ -open set, neutrosophic $\delta\beta$ -closed set, neutrosophic $\delta\beta$ -interior, neutrosophic $\delta\beta$ -closure, neutrosophic $\delta\beta$ -connected space, neutrosophic $\delta\beta$ -separated sets, neutrosophic super $\delta\beta$ -connected space, neutrosophic extremely $\delta\beta$ -disconnected space, neutrosophic strongly $\delta\beta$ -connected space.

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1. Introduction

Many real-life problems in Business, Finance, Medical sciences, Engineering and Social sciences deal with uncertainties. There are difficulties in solving the uncertainties in these data by traditional mathematical models. Neutrosophic system plays important role in the fields of Information Systems, Computer Science, Artificial Intelligence, Applied Mathematics, Mathematics decision making, Medicine, Management Science, and Electrica & Electronic, etc. In 2021, Vadivel et al. introduced neutrosophic δ -pre open set, neutrosophic δ -semi open set, neutrosophic $\delta\alpha$ -open set and $\delta\beta$ -open set in neutrosophic topological spaces. In this paper, the concepts of neutrosophic $\delta\beta$ -connected space, neutrosophic $\delta\beta$ -disconnected space, neutrosophic $\delta\beta$ -separated sets, neutrosophic super $\delta\beta$ -connected space, neutrosophic extremely $\delta\beta$ -disconnected space and

neutrosophic strongly $\delta\beta$ -connected space are discussed in neutrosophic topological spaces. We investigate and study several properties and characterizations concerning connectedness in these spaces.

2. Preliminaries

We recall basic definitions and operations of neutrosophic sets and neutrosophic topological space.

Definition 2.1. Let X be a non-empty fixed set. A neutrosophic set P is an object having the form

$P = \{ \langle x, \mu_P(x), \sigma_P(x), \gamma_P(x) \rangle : x \in X \}$, where $\mu_P(x)$ -represents the degree of membership, $\sigma_P(x)$ -represents the degree of indeterminacy and $\gamma_P(x)$ - represents the degree of non-membership. The class of all neutrosophic sets on X will be denoted by $\mathbb{N}(X)$.

Definition 2.2. Let X be a non-empty set and let $P = \{ \langle x, \mu_P(x), \sigma_P(x), \gamma_P(x) \rangle : x \in X \}$ and

$Q = \{ \langle x, \mu_Q(x), \sigma_Q(x), \gamma_Q(x) \rangle : x \in X \}$ be two neutrosophic sets, Then

1. (Empty set) $0_N = \langle x, 0, 0, 1 \rangle$ is called the neutrosophic empty set.
2. (Universal set) $1_N = \langle x, 1, 1, 0 \rangle$ is called the neutrosophic universal set.
3. (Inclusion): $P \subseteq Q$ if and only if $\mu_P(x) \leq \mu_Q(x)$, $\sigma_P(x) \leq \sigma_Q(x)$ and $\gamma_P(x) \geq \gamma_Q(x) : \forall x \in X$.
4. (Equality): $P = Q$ if and only if $P \subseteq Q$ and $Q \subseteq P$.
5. (Complement): $P^c = 1_N - P = \{ \langle x, \gamma_P(x), 1 - \sigma_P(x), \mu_P(x) \rangle : x \in X \}$.
6. (Union): $P \cup Q = \{ \langle x, \max(\mu_P(x), \mu_Q(x)), \max(\sigma_P(x), \sigma_Q(x)), \min(\gamma_P(x), \gamma_Q(x)) \rangle : x \in X \}$.
7. (Intersection): $P \cap Q = \{ \langle x, \min(\mu_P(x), \mu_Q(x)), \min(\sigma_P(x), \sigma_Q(x)), \max(\gamma_P(x), \gamma_Q(x)) \rangle : x \in X \}$.

Definition 2.3. A neutrosophic point $x_{(\alpha, \beta, \gamma)}$ is said to be in the neutrosophic set A - in symbols $x_{(\alpha, \beta, \gamma)} \in A$ if and only if $\alpha < \mu_A(x)$, $\beta < \sigma_A(x)$ and $\gamma > \gamma_A(x)$.

Definition 2.4. A neutrosophic topology on a non-empty set X is a family \mathbb{T}_N of neutrosophic subsets of X satisfying:

- (i) $0_N, 1_N \in \mathbb{T}_N$.
- (ii) $G \cap H \in \mathbb{T}_N$ for every $G, H \in \mathbb{T}_N$.
- (iii) $\bigcup_{j \in J} G_j \in \mathbb{T}_N$ for every $\{G_j : j \in J\} \subseteq \mathbb{T}_N$.

Then the pair (X, \mathbb{T}_N) is called a neutrosophic topological space. The elements of \mathbb{T}_N are called neutrosophic open sets in X . A neutrosophic set A is called a neutrosophic closed set if and only if its complement A^c is a neutrosophic open set.

Definition 2.5. Let (X, \mathbb{T}_N) be a neutrosophic topological space and A be a neutrosophic set. Then

- (i) The neutrosophic interior of A , denoted by ${}_N \text{Int}(A)$ is the union of all neutrosophic open subsets of A . Clearly ${}_N \text{Int}(A)$ is the biggest neutrosophic open subset of X contained in A .
- (ii) The neutrosophic closure of A denoted by ${}_N \text{Cl}(A)$ is the intersection of all neutrosophic closed sets containing A . Clearly ${}_N \text{Cl}(A)$ is the smallest neutrosophic closed set which contains A .