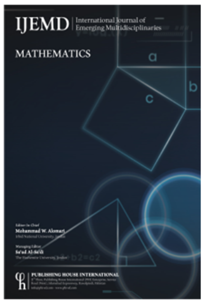
IJEMD-M, 1 (3) (2022) 24 - 39 https://doi.org/10.54938/ijemdm.2022.01.3.88

International Journal of Emerging Multidisciplinaries: Mathematics

Research Paper
Journal Homepage: www.ojs.ijemd.com
ISSN: 2790-1998 (print), 2790-3257 (online)



Flow and Heat Transfer of Power Law Fluid over Horizontal Stretching Cylinder with Partial Slip Condition and Thermal Radiation

Azeem Shahzad[1,*], Zakia Shamim[1] and Tahir Naseem[1,2]

[1]*Department of Mathematics, University of Engineering and Technology, Taxila, 47050, Pakistan*
[2]*Government Degree College Khanpur, Haripur, 22620, Pakistan*

**Corresponding author*

Abstract

The aim of present study is to investigate boundary layer flow of power-law fluid over the horizontal stretching cylinder. The temperature dependent thermal conductivity of the power-law fluid is considered. Combined effects of constant thermal conductivity and viscous dissipation are analyzed in heat transfer. The relevant boundary layer partial differential equations (PDEs) are transformed into ordinary differential equations (ODEs) by using suitable transformations. These nonlinear ordinary differential equations are solved by BVP4C method using MATLAB. The accuracy of computed results is checked by comparing with existing literature. To discuss the effects of flow parameters on velocity and temperature profiles graphs with numeric values are developed. The constant thermal conductivity and radiation effects enhanced corresponding to a higher fluid temperature. The thermal boundary layer thickness enhanced with the increase in Eckert number. The effect of all physical parameters on skin friction coefficient and local Nusselt number are discussed through tables and graphs.

Keywords: Heat Transfer; Power Law Fluid; Partial Slip; Thermal Radiation; Transformation
2010 Mathematics Subject Classification: 35Q79; 74F05

1. Introduction

Water and air, which are referred to be Newtonian fluids, are acknowledged as being the most vital to life. We

Email addresses: azeemhaadi@yahoo.com (A. Shahzad), zakiashamim02@gmail.com (Z. Shamim), tahir.gch@gmail.com)
tahir.gch@gmail.com (T. Naseem)

are acquainted with the flow of Newtonian liquids due to the common nature of water and air. The flow of Newtonian fluids and heat transfer analysis has been the subject of intensive study due to their extensive applications in engineering and industry. In contrast, the expanded uses of fluid flows have pushed scientists and researchers to broaden their study on the non- Newtonian fluids flow and heat transfer. Many industrial, biological, and chemical engineering fluids do not adhere to the Newtonian viscosity relation and are thus referred to as non-Newtonian fluids. Numerous industrial fluids exhibit non-Newtonian behavior, including polymers, nuclear fuel slurries, molten plastics, food, and fossil fuel. Due to its complexity, a single constitutive relation cannot demonstrate all of the properties of non-Newtonian liquids. There are various properties that help us classify these fluids in general.

From an industrial and technical perspective, the most essential property of fluids is their perceived viscosity. Recently, several scientists have found fluids with diverse physical assumptions and geometries. Several researchers have examined the power-law fluid by using the shear rate and viscosity of the non-Newtonian fluid. Zhang [1] studied the impacts of the non-Newtonian power-law fluid's velocity and temperature field in annular flow. Schowalter [2] explains how boundary layer theory may be applied to power-law pseudo plastic-fluids. The non-Newtonian Sisko fluid flow was discussed by Nadeem et al. [3] in an evenly inclined tube. The physical behavior of Sisko fluid passing through an annular pipe was discussed by Khan et al. [4]. The MHD Power- law fluid that flow and heat transfer over a vertically stretching sheet was investigated by Prasad et al. [5]. Hassanien et al. [6] examined the boundary layer problem of flow and heat transfer over stretching surface by using the Power-law model with variable wall temperature.

Megahed et al. [8] presented the impact of heat flux on the steady non-Newtonian power-law fluid over a stretching surface. Mahmoud [9] discovered the impact of surface slip on non-Newtonian fluid by using power-law model. Kannan and Moorthy considered the specific impacts of varying viscosity on power law fluids, slip velocity on a continuous moving surface with non-Newtonian fluids in the situation of suction, and heat generation. [10]. Elfeshawey, A.S. and Waheed [11] studied boundary layer MHD flow by considering Power-law model with thermal radiation flow over a stretching sheet. Malik et al. [12] investigated the influence of an applied magnetic field on the Sisko fluid over-stretching cylinder. The non-Newtonian viscoelastic fluid over a stretching surface was studied by Abel et al. [13]. Patil et al. [14] consider a permeable cylinder and discussed the boundary layer flow over extended sheet with mixed convection boundary conditions. The steady MHD flow of Non-Newtonian fluid was investigated by Prasad et al. [15].

The study of heat transfer across moving surfaces has drawn the interest of contemporary scholars due to its rising demand in science and engineering, particularly chemical engineering. According to these techniques, two factors largely impact the fluid mechanical properties of the finished product. One is the rate of stretching, and the other is the cooling characteristic of the liquid. Numerous experimental conditions necessitate physical qualities with varying fluid properties. By considering stagnation point flow Chiam [16] discussed heat transfer over a stretching surface by considering variable thermal conductivity. Many researchers [17-23] discussed the Non-Newtonian fluid flow and heat transfer by considering variable thermal conductivity and gave a valuable contribution to the literature. Ahmed et al. [24] studied the unsteady axisymmetric flow over stretching surface by considering the power-law model. Abbas et al. [25] developed heat transfer research owing to an extended cylinder with suction and partial slip condition. Krishnamurthy et al. [26] examined the power-law fluid flow generated by a cylinder's expansion. Numerous polymers with a wide variety of shear rates have features that may be characterized by a fluid governed by the power law.

In this article the flow of power-law fluid due to a stretching cylinder is discussed. A power-law fluid tells the properties of many polymers which have wide range of shear rates. The main purpose here is to investigate the boundary layer flow of power-law due to a stretching cylinder. In the present study, constant thermal conductivity is considered. The thermal radiation effects are also presented in addition. The resulting problems for the velocity field and temperature field are first modeled and then solved by the BVP4C method by using MATLAB and the skin friction and the local Nusselt number are examined carefully. The accuracy of computed results is checked by comparing with existing literature.

2. Mathematical formulation

Consider a two-dimensional, unsteady and incompressible flow of a viscous fluid over a permeable nonlinear stretching cylinder in the presence of applied magnetic field. Assume that z-axis is taken along the axis of cylinder and r-axis in radial direction as shown in **Fig.1** Present flow is restricted to z > 0 and z < 0 is due to the simultaneous effect of two equal and opposite forces along the direction of the z-axis with velocity $U(z,t) = \frac{bz}{1-\alpha t}$, where b and α are both positive constants and origin is fixed. The temperature at the cylinder surface is varying as a function of time $T_w(z,t)$ and the ambient fluid temperature is T_∞ , where $T_w > T_\infty$. On base of assumptions of axisymmetric flow, the momentum and energy equation in the cylindrical coordinate are given by:

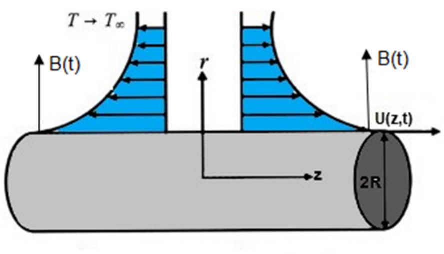


Fig. 1. Schematic Diagram of the Problem.

$$\frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{\partial w}{\partial z} = 0, \tag{1}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = -\frac{k}{\rho}\frac{1}{r}\left[\left(\frac{\partial w}{\partial r}\right)^{n} + n\left(\frac{\partial w}{\partial r}\right)^{n-1}\frac{\partial^2 w}{\partial r^2}\right] - \frac{\sigma B_\circ^2}{\rho(1-\alpha t)}w, \tag{2}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z} = -\frac{k}{\rho c_p}\left(\frac{\partial w}{\partial r}\right)^{n+1} + \frac{k^*(t)}{\rho c_p}\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right] - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial r}, \tag{3}$$

with the boundary conditions;

$$w = U + \lambda\left(\left|\frac{\partial w}{\partial r}\right|^{n-1}\frac{\partial w}{\partial r}\right), u = 0, -k\frac{\partial T}{\partial r} = h_{ext}(T_w - T_\infty), r = R, \tag{4}$$

$$w = 0, T \longrightarrow T_\infty, r \longrightarrow \infty, \tag{5}$$

here z, r are the cylindrical polar coordinates in axial and radial directions, where u and w are the velocity components of fluid in the r and z directions respectively. Whereas n is power law index, k is thermal conductivity, ρ is the particles density of fluid, c_p is specific heat, T_w is the temperature of fluid at wall and T_∞ is the temperature away from the surface of fluid.

$k^*(t) = k_\circ \left|\frac{\partial T}{\partial r}\right|^{n-1}$ is the constant thermal conductivity, q_r is the non- linear radiative heat flux expressed by $q_r = -\frac{4\sigma^*}{3k^\circ}\frac{\partial T^4}{\partial r}$, σ^* is the Stefan-Boltzmann constant and k° is the mean absorption coefficient.

By using the following similarity transformations, the governing flow equations can be reduced into set of ordinary differential equations;

$$\eta = \frac{r^2 - R^2}{2Rz}Re^{\frac{1}{n+1}}, Re = \frac{U^{2-n}z^n}{k/\rho}, \tag{6}$$

$$\psi = RUz\, Re^{\frac{-1}{n+1}} f(\eta), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},$$

whereas, η is the independent variable and θ is the non-dimensional temperature. The stream function $\psi(r, z)$ of fluid distribution is defined as;

$$u = -\frac{1}{r}\frac{\partial \psi}{\partial z}, w = \frac{1}{r}\frac{\partial \psi}{\partial r}, \tag{7}$$

By using the similarity transformations from Eq. (6) in Eqs. (1-3). The continuity equation is identically satisfied, while momentum and heat equations Eq. (2-3) are transformed into nonlinear ordinary differential equations;

$$A\left[f' + \eta f''\left(\frac{2-n}{n+1}\right)\right] - \frac{2n}{n+1}ff'' + f'^2 + (n+1)C(1+2\eta C)^{\frac{n-1}{2}}f''^n + n(1+2\eta C)^{\frac{n+1}{2}}f''^{n-1}f''' - Mf' = 0 \tag{8}$$

$$PrA\left[\theta + \left(\frac{2-n}{1+n}\right)\eta\theta'\right] + Pr\left[f\theta' - \frac{2n}{n+1}\theta' f\right] - \left[\theta''\left(-\theta'^{n-1}\right)(1+2\eta C)^{\frac{n+1}{2}}\right]\left(1+\frac{4}{3}R\right) + EcPr(-f'')^{n+1}(1+2\eta C)^{\frac{n+1}{2}} + \left[2C(1+2\eta C)^{\frac{n-1}{2}}(-\theta)^n\right]\left(1+\frac{4}{3}R\right) = 0 \tag{9}$$

$$f(0) = 0, f'(0) = 1 - \lambda_1\left(-f''(0)\right)^n, \theta'(0) = -\gamma\left(1-\theta(0)\right) \tag{10}$$

$$f'(\infty) = 0\,, \theta(\infty) = 0$$

where, $A = \frac{\alpha}{b}$ is the unsteadiness parameter, $C = \frac{z}{R}Re^{\frac{-1}{n+1}}$ is the curvature parameter, $M = \frac{\sigma B_\circ}{\rho b}$ is the magnetic parameter. $Pr = \frac{k}{k_\circ}c_p b^{1-n}C_\circ^{\,n-1}$ is the Prandtl number, $Ec = \frac{U^2}{c_p(T_w - T_\infty)}$ is the Eckert number, $R = \frac{4T_\infty^3\sigma^*}{\rho c_p k^\circ k^*}$ is the radiation parameter, $\lambda_1 = \frac{\lambda U^{n-1}}{z^n}Re^{\frac{n}{n+1}}$ is the skin parameter and

$\gamma = \frac{-h_{ext}}{k}z\,Re^{\frac{-1}{n+1}}$ is the surface convection parameter. The local skin friction coefficient and the wall heat transfer coefficient (Nusselt number) are the physical quantities that are:

$$C_f = \frac{2\tau_w}{\rho U_w^2}\,, \qquad Nu = \frac{zq_w}{k(T_w - T_\infty)}$$

$$\tau_w = k\left(\left|\frac{\partial w}{\partial r}\right|^n\right)_{r=R}, \qquad q_w = -k\left(1+\frac{16\sigma^* T_\infty^3}{3k^\circ k^*}\right)\left(\frac{\partial T}{\partial r}\right)_{r=R} \tag{11}$$

The local skin friction coefficient and the wall heat transfer coefficient (Nusselt number) Eq. (11) after converting into dimensionless form as;

$$\frac{1}{2}C_f\,Re^{\frac{1}{n+1}} = -[-f''(0)]^n\,, \qquad Nu\,Re^{\frac{-1}{n+1}} = -\left(1+\frac{4}{3}R\right)\theta'(0). \tag{12}$$

4. Solutions of the problem

To solve the system of non-linear equations, many researchers proposed different techniques, such as shooting, Kellor box and finite difference methods. Here, we use BVP4C technique to determine the solution of non-linear system of equations. BVP4C scheme can be viewed as an implicit Runge Kutta formula with a continuous extension. The basic method of BVP4C is a well-known Simpson's method, and is run into different codes.

Further, the detailed discussion on BVP4C along with convergence and error analysis can be found in [22,23]. The different Charteris of the proposed BVP4C scheme, such as faster convergence with lesser error, accept not only two-point but multi-point BVPs with better accuracy, and dealing with singular BVPs etc. due to which it become much popular in research [24,25].

The numerical solution of a nonlinear system of equations (6-8), is find by the set of first-order linear equations by considering:

$$f = y_1, f' = y_2, f'' = y_3, f''' = y'_3$$

$$\theta = y_4, \theta' = y_5, \theta'' = y'_5$$

$$y_1(0) = 0, y_2(0) = 1 - \lambda_1(-y_3(0))^n, \; y_5(0) = -\gamma(1 - y_4(0)) \tag{13}$$

$$y_2(\infty) = 0, \; y_4(\infty) = 0$$

$$y_3' = A\left[y_2 + \eta y_3\left(\frac{2-n}{n+1}\right)\right] - \frac{2n}{n+1}y_1y_3 + y_2^2 + (n+1)C(1+2tC)^{\frac{n-1}{2}}y_3^n \div n(1+2tC)^{\frac{n+1}{2}}y_3^{n-1} \tag{14}$$

$$y'_5 = PrA\left[y_4 + \left(\frac{2-n}{1+n}\right)ty_5\right] + Pr\left[y_1y_5 - \frac{2n}{n+1}y_5y_1\right] - EcPr(-y_3)^{n+1}(1+2tC)^{\frac{n+1}{2}} + 2C(1+2tC)^{\frac{n+1}{2}}(-y_5)^n\left(1+\frac{4}{3}R\right) \div \left(1+\frac{4}{3}R\right)(1-2tC)^{\frac{n+1}{2}}(y_5)^{n-1}. \tag{15}$$

Finally, the first-order ODEs along with boundary conditions are obtained. It is done by hit and trail method. To validate the results obtained we computed numerical values of $f''(0)$ and $\theta'(0)$ are compared with some previous studies under some limiting cases. Table 3 present the value of $f''(0)$ compared with that Mishra *et.al*, Abel *et.al* and Anderson case by taking $M = 0, C = 0, A = 0, k = 0$ that tells the validity, accuracy and effectiveness of the proposed scheme. Moreover, the given results ensure the advantage of BVP4C scheme.

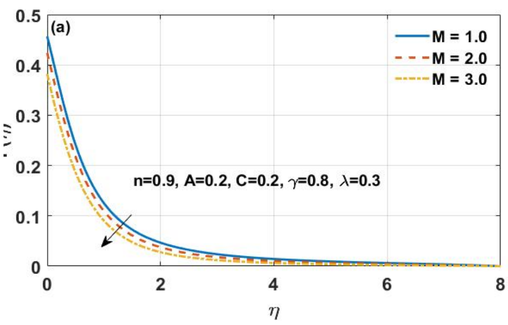
5. Numerical results and discussion

Our intention in this section is to analyze the velocity and temperature profiles for different physical parameters, e.g., the curvature parameter C, the magnetic parameter M, unsteadiness parameter A the Eckert number Ec, the Prandtl number Pr, the radiation parameter R, the variable thermal conductivity parameter k, slip parameter λ_1 and convection parameter γ. For illustrations of the results, numerical values are plotted in Figs. 2-10**.** The values of the skin friction coefficient and the Nusselt number are calculated and presented in Table 2 and Table 3 respectively.

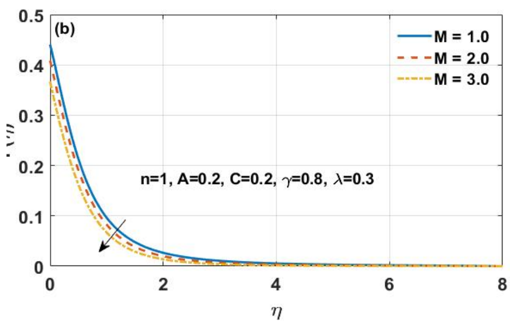
Fig. 2 represents the effects of *M* on the velocity profile. It is noted that the increasing values of magnetic parameter reduce the velocity profile. The figure clearly shows that applied magnetic field opposes the fluid velocity. This is due to the fact that variation of M leads to the variation of the Lorentz force induced by the magnetic field. Magnetic field on electrically conducting fluid produced the resistance force known as Lorentz force. It makes higher limitation to the fluids which resist the flow lessening fluid velocity. Applied magnetic field causes, the fluid to polarize and form dipoles. Due to which the flow restricted. From Fig. 3 it is clear that the curvature parameter C and velocity has direct relation. When velocity boosts value of C increased. It is due to the radius of cylinder contracted when the value of C increases due to which less interaction with surface area producing less resistance regarding the particles of fluid. It is also clear from Fig. 3 that curvature parameter C decreases the velocity profile near the wall of cylinder and it increases away from the wall.

Table 1: Values of skin friction $-f''(0)$ for various values of power-law index n with $M = 0$

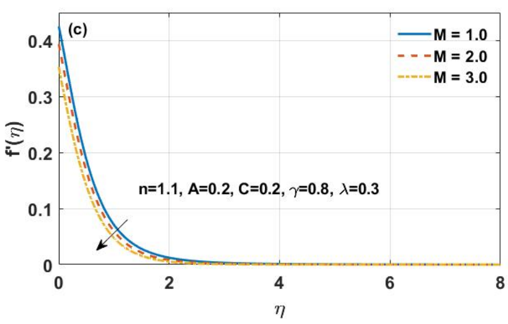
n	Anderson [26]	Abel *et.al* [13]	Mishra *et.al* [27]	Present study
0.2	1.9287	1.94369	1.94444	1.96266
0.5	1.1605	1.16774	1.16712	1.17760
1	1.0000	1.0000	1.0000	1.0000
1.2	0.9874	0.98737	0.98737	0.98723
1.5	0.9806	0.98065	0.98057	0.98055



(a)

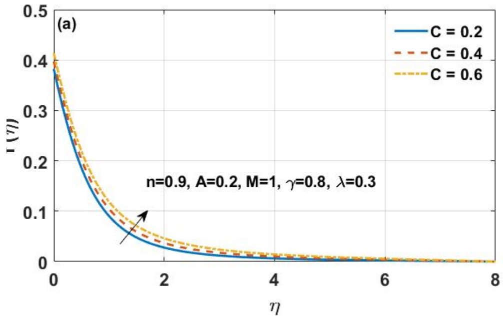


(b)

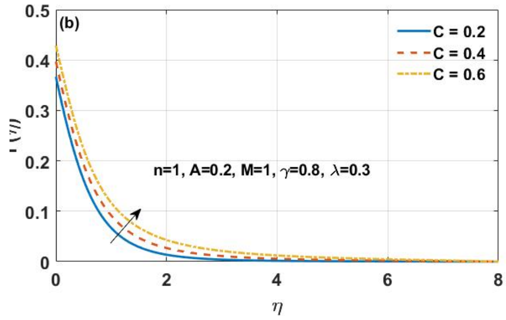


(c)

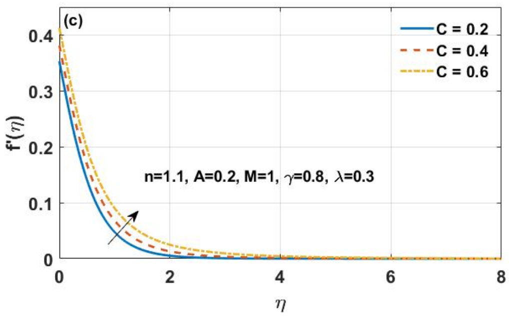
Fig. 2 (a-c). Variation of M on velocity profile f'



(a)



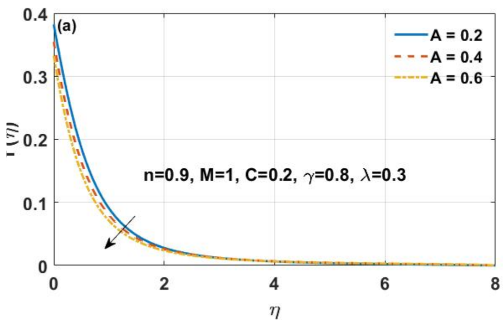
(b)

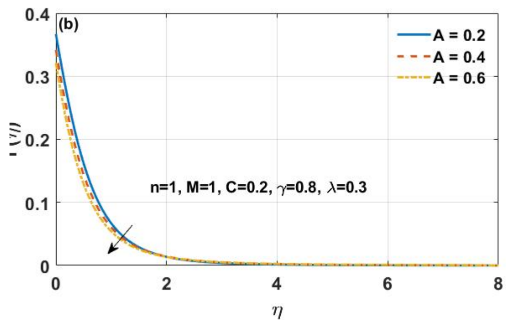


(c)

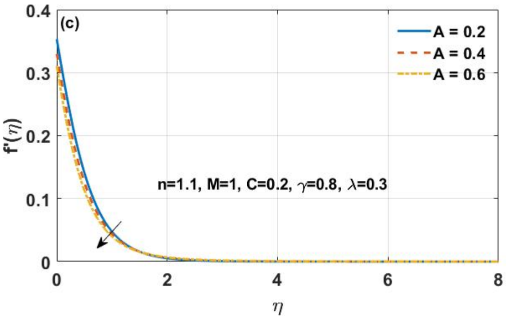
Fig. 3 (a-c). Variation of C on velocity profile f'

In Fig. 4 unsteadiness parameter on velocity profile is examined. It is observed that increasing the unsteady parameter reduces the velocity profiles and this effect is accompanied by a reduction in the momentum boundary layer thickness in velocity profiles which indicates that the unsteadiness parameter reduces the flowrate due to the stretching cylinder. Fig. 5 shows that with the increase in λ_1 such slip velocity increases and consequently the fluid velocity decreases because under the slip condition, the pulling of the stretchable cylinder can only be partly transmitted to the fluid. Thus, the increasing values of the slip parameter reduce the dimensionless velocity profile.



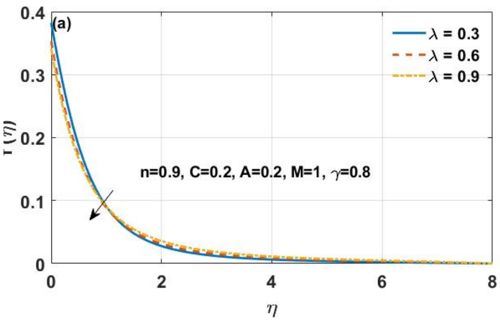


(a) (b)

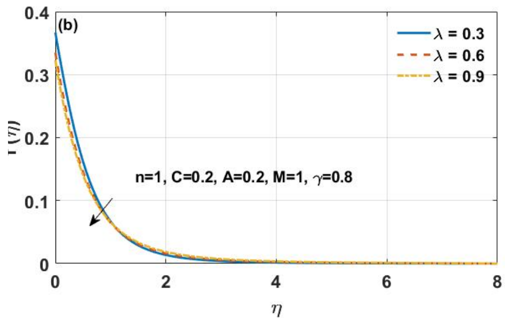


(c)

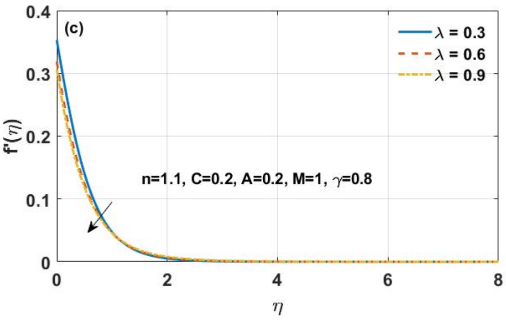
Fig. 4 (a-c). Variation of A on velocity profile f'



(a)



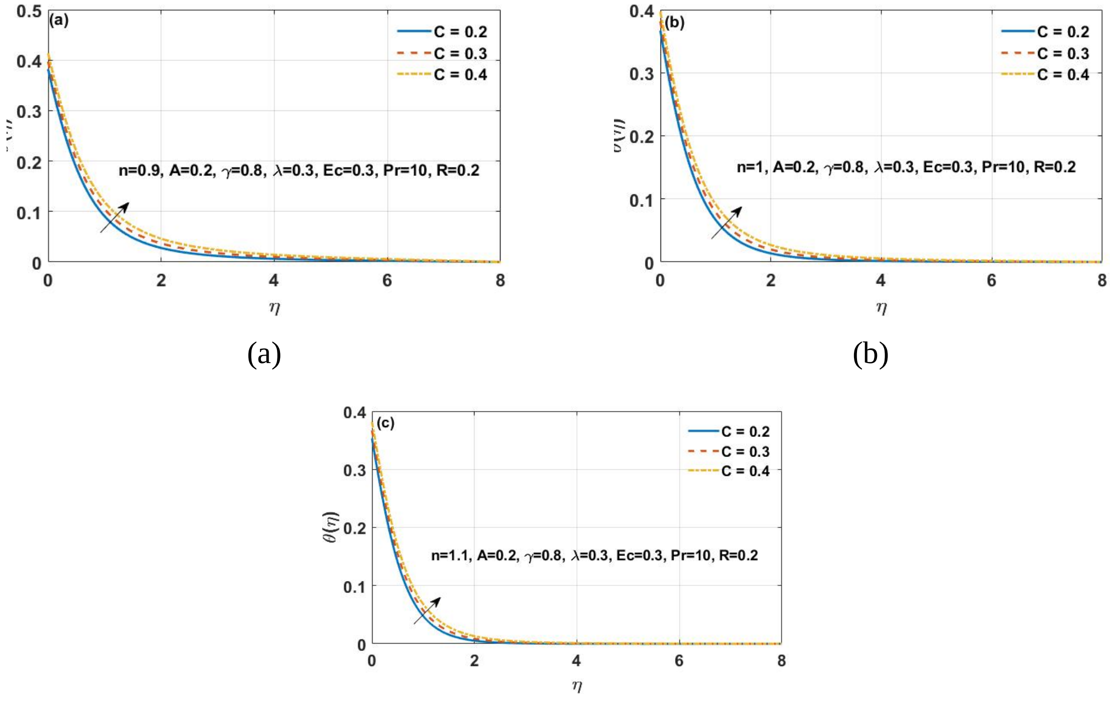
(b)



(c)

Fig. 5 (a-c). Variation of λ on velocity profile f'

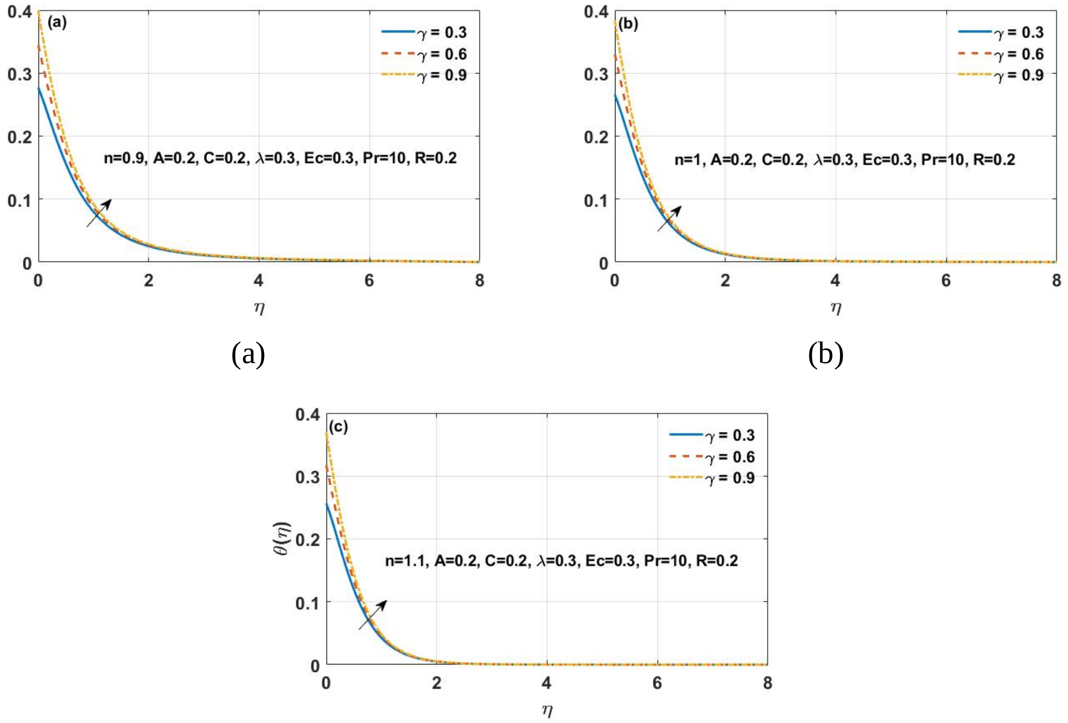
In Fig. 6 It can be seen that C increases the radius of curvature reduces enhancing the velocity due to which K.E of the molecules increases causing increase in temperature. Fig. 7 indicates the convection parameter γ is directly proportional to the heat transfer coefficient associated with the hot fluid h_{ext} . The thermal resistance on the hot fluid side is inversely proportional to h_{ext} . Thus increases, the hot fluid side convection resistance decreases and consequently, the temperature profile increases.



(a) (b)

(c)

Fig. 6 (a-c). Variation of C on temperature profile θ

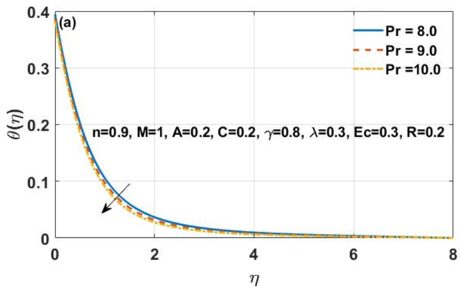


(a) (b)

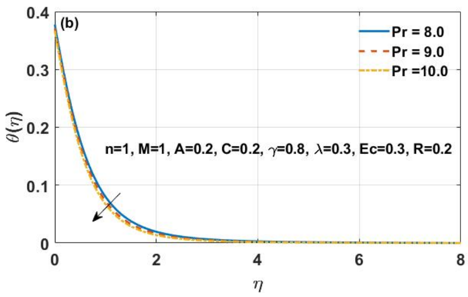
(c)

Fig. 7 (a-c). Variation of γ on temperature profile θ

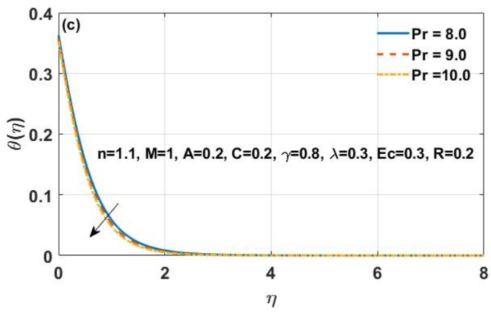
It is clear from Fig. 8 that when the values of Pr increases reduce the temperature profile. Fluids which have lower Prandtl numbers have higher thermal conductivities (and thicker thermal boundary layer) that's why heat can diffuse from the cylinder faster than form higher Pr fluids (thinner boundary layers). So, the Prandtl number is used in conductive fluids to increase the rate of cooling. In Fig. 9 graphs shows the temperature distribution against the Eckert number Ec . Basically, the Eckert number relies on the K.E. When the values of the Eckert number increase, the K.E enhances. This increment in the kinetic energy causes the increases in the temperature and thermal boundary layer thickness. In all these cases, the velocity is the maximum at the surface of the cylinder, and it starts decreasing away from the surface of cylinder. The increase in thermal radiation R speeds the temperature and the boundary layer thickness Fig. 10. The temperature profile increases with the increase in R, because larger in R implies higher surface heat flux and then the fluid become warmer.



(a)

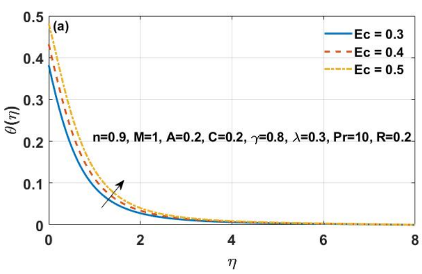


(b)

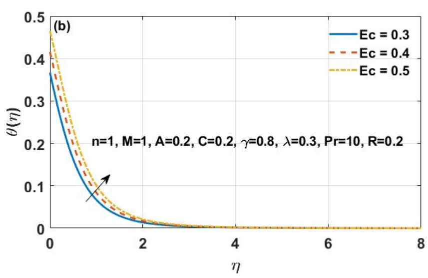


(c)

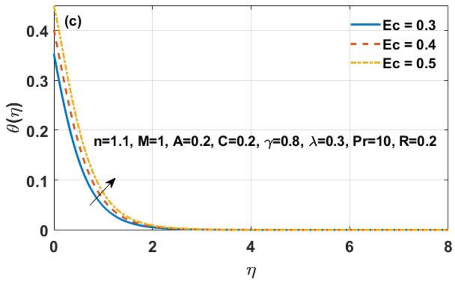
Fig. 8 (a-c). Variation of Pr on temperature profile θ



(a)

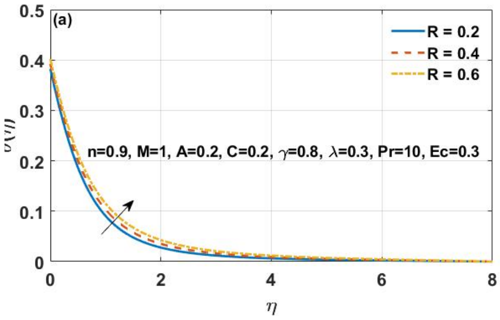


(b)

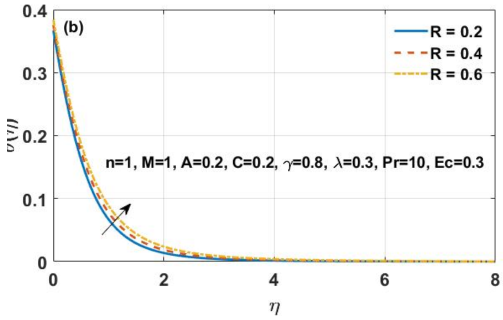


(c)

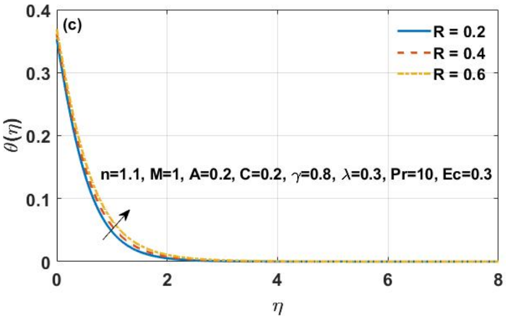
Fig. 9 (a-c). Variation of Ec on temperature profile θ



(a)



(b)



(c)

Fig. 10 (a-c). Variation of R on temperature profile θ

Table 2: Values of skin friction $-(-f''(0))^n$ for various values of physical parameters

	Physical parameters				$-[-f''(0)]^n$	
M	C	A	λ	$n = 0.9$	$n = 1.0$	$n = 1.1$
1	0.2	0.2	0.3	1.0505216	1.0566712	1.0630225
2	-	-	-	1.1813123	1.1976386	1.2133663
3	-	-	-	1.2793083	1.3031775	1.325812
1	0.2	-	-	1.0505216	1.0566712	1.0630225
-	0.4	-	-	1.1407206	1.154820	1.1685211

-	0.6	-	-	1.2335251	1.257334	1.2799576
-	0.2	0.2	-	1.0505216	1.0566712	1.0630225
-	-	0.4	-	1.0708216	1.0799257	1.0891448
-	-	0.6	-	1.090443	1.1022341	1.1140588
-	-	0.2	0.3	1.0505216	1.0566712	1.0630225
-	-	-	0.6	0.79722178	0.78732127	0.7786333
-	-	-	0.9	0.64390325	0.63049296	0.6187231

Table 3: Values of Nusselt number $-\theta'(0)$ for various values of physical parameters

Physical Parameters					$-\theta'(0)$		
C	γ	Ec	Pr	R	$n = 0.9$	$n = 1.0$	$n = 1.1$
0.2	0.8	0.3	10	0.2	0.49412023	0.50653371	0.51750271
0.3	-	-	-	-	0.48198627	0.49483933	0.5063226
0.4	-	-	-	-	0.4692640	0.48248722	0.49447281
0.2	0.3	-	-	-	0.21687714	0.22024724	0.22307299
-	0.6	-	-	-	0.39319433	0.40202111	0.40973493
-	0.9	-	-	-	0.54047383	0.55459247	0.56710993
-	0.8	0.3	-	-	0.49412023	0.50653371	0.51750271
-	-	0.4	-	-	0.45444281	0.46740392	0.47888479
-	-	0.5	-	-	0.41476996	0.42827412	0.44027531
-	-	0.3	8	-	0.48458279	0.49794351	0.50978435
-	-	-	9	-	0.48980442	0.50265839	0.5140304
-	-	-	10	-	0.49412023	0.50653371	0.51750271
-	-	-	-	0.2	0.49412023	0.50653371	0.51750271
-	-	-	-	0.4	0.48604731	0.49926862	0.51097986
-	-	-	-	0.6	0.47841268	0.49233905	0.50471054

5. Concluding Remarks

This paper presents an investigation on the flow and heat transfer in the presence of constant thermal conductivity and thermal radiation effects on power-law fluids over a stretching cylinder. The momentum equations and the heat equation are reduced to two non-linear ordinary differential equations (ODEs) by suitable similarity transformations. The solutions are found out by using the BVP4C scheme. Some of the important results of this research paper are as follows:

- The increment in curvature parameter C decreases the velocity profile near the wall of cylinder and increases it far away from the wall. By enhancing magnetic parameter M and slip parameter λ velocity of fluid decreases.
- The temperature profile $\theta(\eta)$ increases when curvature parameter C , magnetic parameter M , Eckert

number Ec , thermal radiation R and convection parameter γ increases with different values of n, on the other hand fluid temperature $\theta(\eta)$ decreases by increasing Prandtl number Pr.

- The skin friction coefficient extends when increasing estimations of magnetic parameter, curvature parameter and unsteadiness parameter.
- Curvature parameter C and Prandtl number Pr increases the values of local Nusselt number. While, Nusselt number declines for larger values of Eckert number Ec and radiation parameter R.

References

[1] Zhang, H., Kang, Y. and Xu, T. Study on Heat Transfer of Non-Newtonian Power Law Fluid in Pipes with Different Cross Sections. *Procedia engineering*, **205**, 3381-3388 (2017).

[2] Schowalter, W.R. The application of boundary-layer theory to power-law pseudoplastic fluids: Similar solutions. *AIChE Journal*, **6**(1), 24-28 (1960).

[3] Nadeem, S. and Akbar, N.S. Peristaltic flow of Sisko fluid in a uniform inclined tube. *Acta Mechanica Sinica*, **26**(5), 675-683 (2010).

[4] Khan, M., Munawar, S. and Abbasbandy, S. Steady flow and heat transfer of a Sisko fluid in annular pipe. *International Journal of Heat and Mass Transfer*, **53**(7-8), 1290-1297 (2010).

[5] Prasad, K.V., Datti, P.S. and Vajravelu, K. Hydromagnetic flow and heat transfer of a non-Newtonian power law fluid over a vertical stretching sheet. *International Journal of Heat and Mass Transfer*, **53**(5-6), 879-888 (2010).

[7] Hassanien, I.A., Abdullah, A.A. and Gorla, R.S.R. Flow and heat transfer in a power-law fluid over a nonisothermal stretching sheet. *Mathematical and computer modelling*, **28**(9), 105-116 (1998).

[8] Megahed, A.M. Flow and heat transfer of a non-Newtonian power-law fluid over a non-linearly stretching vertical surface with heat flux and thermal radiation. *Meccanica*, **50**(7), 1693-1700 (2015).

[9] Mahmoud, M.A. Slip velocity effect on a non-Newtonian power-law fluid over a moving permeable surface with heat generation. *Mathematical and Computer Modelling*, **54**(5-6), 1228-1237 (2011).

[10] Kannan, T. and Moorthy, M.B.K., 2016. Effects of variable viscosity on power-law fluids over a permeable moving surface with slip velocity in the presence of heat generation and suction. *Journal of Applied Fluid Mechanics*, **9**(6), 2791-2801.

[11] Elfeshawey, A.S. and Waheed, S.E. Effect of viscous dissipation and thermal radiation on MHD flow and heat transfer for a power-law fluid with variable fluid properties over a permeable stretching sheet. *Waves in Random and Complex Media*, 1-15 (2022).

[12] Malik, M.Y., Hussain, A., Salahuddin, T., Awais, M. and Bilal, S. Numerical solution of Sisko fluid over a stretching cylinder and heat transfer with variable thermal conductivity. *Journal of Mechanics*, **32**(5), 593-601 (2016).

[13] Abel, M.S., Tawade, J. and Nandeppanavar, M.M. Effect of non-uniform heat source on MHD heat transfer in a liquid film over an unsteady stretching sheet. *International Journal of Non-Linear Mechanics*, **44**(9), 990-998 (2009).

[14] Patil, P.M., Roy, S. and Chamkha, A.J. Mixed convection flow over a vertical power-law stretching sheet. *International Journal of Numerical Methods for Heat & Fluid Flow*, (2010).

[15] Prasad, K.V., Datti, P.S. and Vajravelu, K. Hydromagnetic flow and heat transfer of a non-Newtonian power law fluid over a vertical stretching sheet. *International Journal of Heat and Mass Transfer*, **53**(5-6), 879-888 (2010).

[16] Chiam, T.C. Heat transfer with variable conductivity in a stagnation-point flow towards a stretching sheet. *International Communications in Heat and Mass Transfer*, **23**(2), 239-248 (1996).

[17] Vyas, P. and Rai, A. Radiative flow with variable thermal conductivity over a non-isothermal stretching sheet in a porous medium. *Int. J. Contemp. Math. Sciences*, **5**(54), 2685-2698 (2010).

[18] Naseem, T., Fatima, U., Munir, M., Shahzad, A., Kausar, N., Nisar, K.S., Saleel, C.A. and Abbas, M. Joule heating and viscous dissipation effects in hydromagnetized boundary layer flow with variable temperature. *Case Studies in Thermal Engineering*, **35**, 102083 (2022).

[19] Naseem, T., Nazir, U., Sohail, M., Alrabaiah, H., Sherif, E.S.M. and Park, C. Numerical exploration of thermal transport in water-based nanoparticles: A computational strategy. *Case Studies in Thermal Engineering*, **27**, 101334 (2021).

[20] Malik, M.Y., Hussain, A., Salahuddin, T., Awais, M., Bilal, S. and Khan, F. Flow of Sisko fluid over a stretching cylinder and heat transfer with viscous dissipation and variable thermal conductivity: a numerical study. *AIP Advances*, **6**(4), 045118 (2016).

[21] Naseem, T., Nazir, U., El-Zahar, E.R., Algelany, A.M. and Sohail, M. Numerical Computation of Dufour and Soret Effects on Radiated Material on a Porous Stretching Surface with Temperature-Dependent Thermal Conductivity. *Fluids*, **6**(6), 196 (2021).

[22] Mahmood, A., Ahmed, S. and Iram, H. Joule and Viscous Dissipation Effects on MHD Boundary Layer Flow over a Stretching Sheet with Variable Thickness. *International Journal of Emerging Multidisciplinaries: Mathematics*, **1**(2), 1-10 (2022).

[23] Hussain, A., Afzal, A. and Mahmood, R. Numerical Investigation of Viscous Fluid Flow and Heat Transfer in the Closed Configuration Installed with Baffles. *International Journal of Emerging Multidisciplinaries: Mathematics*, **1**(2), 49-57 (2022).

[24] Ahmed, J., Begum, A., Shahzad, A. and Ali, R. MHD axisymmetric flow of power-law fluid over an unsteady stretching sheet with convective boundary conditions. *Results in Physics*, **6**, 973-981 (2016).

[25] Abbas, Z., Rasool, S. and Rashidi, M.M. Heat transfer analysis due to an unsteady stretching/shrinking cylinder with partial slip condition and suction. *Ain Shams Engineering Journal*, **6**(3), 939-945 (2015).

[26] Andersson, H.I. Film flow of power-law fluids. *Encyclopedia of Fluid Mechanics*, **9**, 617-648 (1990).

[27] Mishra, S.R., Baag, S., Dash, G.C. and Acharya, M.R. Numerical approach to MHD flow of power-law fluid on a stretching sheet with non-uniform heat source. *Nonlinear Engineering*, **9**(1), 81-93 (2020).