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Multiple Attribute Decision Making Based on Interval-Valued Neutrosophic Trapezoidal Fuzzy Numbers and its Application in the Diagnosis of Viral Flu

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Abstract

Decision-making technique (DMT) is mostly used in artificial intelligence and cognitive sciences to elaborate individual and social perception. So, one of the most important strategies in DMT evolved in medical diagnosis scrutiny regarding the connection of symptoms and diagnosis of diseases due to uncertainty and fuzziness in the relevant information. The focus of this article is to develop a diagnostic decision making strategy for the diagnosis of Viral diseases with close related symptoms using the Interval-valued trapezoidal neutrosophic fuzzy Numbers (IVTrNFN) w.r.t multiple attribute decision making (MADM) strategy where, the attribute value is evolved to Interval-valued trapezoidal neutrosophic fuzzy number and the attribute weight is unknown and can be related to the GRA (Grey relational analysis projection) technique. In this research several operational laws are developed as well as the expected value and the hamming distance between two IVTrFNs are introduced. Moreover, the information entropy method is used to determine the attributes weights and the grey relational analysis as well as the projection method are involved too in the proposed framework. The ranks of the alternative decisions are evaluated by their relative closeness to PIS (Positive Ideal Solutions), which combine the grey relational projection values from positive and negative ideal solutions associated with each alternative. Finally, a Viral disease example is given to verify the developed approach and to demonstrate its practicality and effectiveness.

Keywords: *Multi-criteria decision making technique (MCDMT); Interval-valued trapezoidal neutrosophic fuzzy number (IVTrNFN); Entropy Technique (ET); Grey Relational Projection Technique (GRPT); Medical Diagnosis.*
2010 Mathematics Subject Classification: *06Dxx; 06D72.*

1. Introduction

In the real world many decision making problems arise in the different fields of life. To tackle or cope up with all such problems we should take an optimal decision. Multi-attribute decision making technique (MADMT) is the best tool to tackle such real life situation problems. In the field of medical sciences, different Viral diseases have many common symptoms due to which it is often difficult to successfully diagnose a certain disease. Zadeh [1] has put forward the fuzzy set theory, which has proved to be important in tackling uncertain situations and fuzziness in real world problems. In 1989 Atanassov andGargov[4] gave extension to the idea of fuzzy theory by putting forward the theory of intuitionistic fuzzy set (IFS) and interval valued intuitionistic fuzzy set (IVIFS). the idea of intuitionistic fuzzy set (IFS) and interval valued intutionistic fuzzy set (IVIFS) which are considered even better than fuzzy sets to capture the uncertain situations. Adlassing [2] used the fuzzy set theory to discuss the connection among medical and the fuzzy logic and explain computerized diagnosing strategy. Touqeer et al. [21, 23, 22] also presented different techniques regarding MCDM using different types of fuzzy sets. De et al[9] put forward the tool of medical diagnosis using intuitionistic fuzzy sets. Abdullah et al.[5] as well as Garg et al. [6, 7, 8, 15, 18] discussed several decision making problems employing fuzzy and intuitionistic fuzzy sets environment.

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In 1995 Florentin Smarandache[14] collected, presented, published and brought into attention the introductory research material for the new concept of Neutrosophy. ”Neutrosophic” set is an important tool through which we can perceive and explain the hierarchy of the ”classic set, fuzzy set ,interval valued fuzzy set , intuitionistic fuzzy set” etc. In the field of Medicare With passage of time the symptoms of different diseases change according to the different individuals infected.Therefore, there is need to develop and adapt sophisticated tools for proper patient diagnosis.

Ye et al.[16, 17, 19] brought into attention the different multi-procedure techniques based on the Neurotrophic equation. In Neutrosophy the employment of different fuzzy sets introduces helpful solutions for the the real world demanding Medical diagnosis situation[16, 17, 19, 20, 24]. In the herein research the GRA (grey relational analysis) introduced by Deng[25, 26] and widely used for MCDM is involved in the proposed framework. Zhang[32] presented GRA method for MCDM with interval numbers. This method has a practical and theoretical appeal in estimating under uncertainty the correlation degree of factors. Accordingly, this theory is based on and, also, leads to supporting the idea that the more the similarity between factors the more these factors correlation.

In the previously mentioned literature as well as in the relevant literature about the GRA methodology, as for instance [25, 26, 27, 32], the different researchers have involved only the IFS (Intuitionistic Fuzzy Sets), the ITFS (Intuitionistic Trapezoidal Fuzzy Sets)[27] and the Interval Valued Intuitionistic Fuzzy Sets (IVIFS[35, 51]. IFS, ITFS and the IVIFS can tackle efficiently incomplete only information but not indeterminate and inconsistent information. Tne Neutrosophic fuzzy set concept is able to generalize the above GRA schemes in decision making. Neutrosophic set theory, integrating the three major degrees of uncertainty in the analysis of information, that is the degree of truth , the degree of indeterminacy and the degree of falsity describes efficiently all the aspects involved and needed in any successful decision making technique applied under uncertainty. Let’s suppose an online product review system where we often make use of statements like ”I said this shoes pair looks fine at first look but I don’t think it looks fine after a long time” and let’s suppose a statistical examination of all such reviews is conducted. Then, the support degree will be 0.8 ,the negation degree could be 0.4 and the uncertainty degree would be 0.3, using the Neutrosophic number approach. If we look closely, the total degree of fuzziness in such uncertain information management is (0.8+0.4+0.3), which is greater than 1, something affordable in NN (Neutrosophic Numbers). Such rich information representation, however, is not possible under the framework of IFS and ITFS representation.

In the herein proposed framework it is attempted to fill in this research gap in literature. That is, to investigate the use of Neutrosophic rich information representation framework in GRA (Grey Relational Analysis) based Decision Making, with emphasis to Medical Decision Making, aiming at improving GRA methodology. More specifically, we are using the interval-valued trapezoidal neutrosophic fuzzy numbers (IVTrNFN) in a (MADMT) model. The difficulties in the decision making process, due to single-valued uncertainty and membership functions of IFS, ITFS can be overcome with the vector valued (T,F,I), standing for truth, falsity, indeterminacy. membership function in neutrosophic situation. It is much more convenient to express fuzziness in (T,F,I) vector valued memberships using interval numbers than involving real crisp numbers, standing for the single valued membership functions. In the IVTrNFN representation the values will be given in the form of intervals rather than in the form of exact numbers.

Therefore, it is herein considered IVTrNFN neutrosophic membership representation, rather than the single valued neutrosophic number representation. Thus, the herein proposed research has investigated the GRA methodology within the newly developed scope of ”Neurotrophic set” theory using the IVTrNFN representation. In neutrosophic set theory, indeterminacy is quantified explicitly and truth-membership, indeterminacy-membership and falsity-membership are modelled as independent. Neutrosophic logic clearly distinguishes between relative truth and absolute truth while fuzzy logic does not provide such rich representations capabilities. Involving the IVTrNFN representational model more robust inference capabilities are provided, rather than the application of rival previous techniques. It is well known that in medical decision making field large amount of complex knowledge is employed and, therefore, the whole process involves low intra and inter person consistency. So, contradictions, inconsistency, indeterminacy and fuzziness should be accepted as unavoidable as it is integrated in the behavior of biological systems as well as in their characterization and decision processes. The IVTrNFN representational model fits very well in the medical decision making processes because in the IVTrNFN model the memebership values are given in the vectored interval numbers format rather then in the exact single membership values format. Using the IVTrNFN representation helps us to explain in more meaningful ways the differences between alternative decision making strategies rather than with the exact values based IFS and ITFS representations. Therefore, generalizing GRA methodology, integrating it with the IVTrNFN representation framework, offers much richer capabilities to deal successfully with the Multi-Criteria Decision Making (MCDM) and Multi-Attribute Decision Making (MADM) real world complex problems, better tackling uncertainty and fuzziness inherent in such problems. Therefore, the proposed generalized GRA methodology framework fits very well in solving complex real world decision making problems like medical decision making. This is exactly the motivation of the proposed approach, conducting the herein research to contribute in demonstrating its rich representation and efficient solution capabilities and its advantages.

The paper outline is as follow. Next section reviews the most important basics of neutrosophic Sets. Section 2, also, introduces ITVrNFNs and explains their relevant arithmetic operations. Section 3 presents a novel strategy for solving Multi-Attribute Decision Making (MADM) problems involving the novel GRA methodology based on the IVTrNFN representation. Section 4 illustrates a relevant real world numerical medical decision making problem and the associated application of the proposed framework, providing an efficient solution, as well as its explanation and practicality. Section 5 provides a qualitative comparative methodological discussion on the proposed and rival approaches and finally, section 6 provides conclusive remarks and prospects.

2 Basic Definition

Neutrosophic Sets
Neutrosophic set theory is a section of neutrosophy, which studies the origin, nature and the scope of neutralities, as well as the interaction with different ideational spectra [14] and consists a powerful formal framework.

Definition 2.1. [28] Assume the X be the sets of points and $x \epsilon X$. Then, the neutrosophic set A in X is defined as $\dot{A} = \{< x, \dot{T}(x), \dot{I}(x), \dot{F}(x) x :\in X\}$ where, $\dot{T}(x) + \dot{I}(x) + \dot{F}(x) \in [0, 1]$ and the $\dot{T}(x)$is the truth- membership grade, while $\dot{I}(x)$ is the indeterminacy grade and the

$\dot{F}(x)$ is the falsity grade.

Definition 2.2. [30] An interval $A(x) = [A^U, A^L]$ represents the (IVFS) defined on universe X where, A^L is the lower fuzzy set $A^L : X \to [0,1]$ and A^U is the upper fuzzy set

$$A^U : X \to [0,1]$$
$$A = \{(x, [A^U, A^L]) : x \in X\}\ 0 \le A^L \le A^U \le 1$$

Definition 2.3. An interval valued trapezoidal neutrosophic fuzzy number($IVTrNFN$) is an interval valued neutrosophic set ($IVTrNS$) on X defined by $N(x) = \{N^L, N^U\}$ where, N^U and N^L are the lower and upper trapezoidal neutrosophic sets of N such that $N^L \subseteq N^U$

Definition 2.4. If N^U is considered to be an Upper (IVTrNFN) then, we have these following three trapezoidal neutrosophic fuzzy numbers as follows, $\widetilde{T}^U = (\bar{\dot{l}}, \bar{\dot{m}}, \bar{\dot{n}}, \bar{\dot{p}}) : X \to [0,1]$, $\widetilde{I}^U = (\bar{\dot{q}}, \bar{\dot{r}}, \bar{\dot{s}}, \bar{\dot{t}}) : X \to [0,1]$ and $\widetilde{T}^U = (\bar{\dot{u}}, \bar{\dot{v}}, \bar{\dot{w}}, \bar{\dot{x}}) : X \to [0,1]$. If $\bar{\dot{m}} = \bar{\dot{n}}$, $\bar{\dot{r}} = \bar{\dot{s}}$ and $\bar{\dot{v}} = \bar{\dot{w}}$. These IVTrNFN are reduced to a triangular neutrosophic fuzzy number to be defined as

$$\widetilde{T}^U = \left\{ \begin{array}{ll} \dfrac{x - \bar{\dot{l}}}{\bar{\dot{m}} - \bar{\dot{l}}} & \text{for } \bar{\dot{l}} \le x < \bar{\dot{m}} \\ \widetilde{T}^U, & \text{for } \bar{\dot{m}} \le x \le \bar{\dot{n}} \\ \dfrac{\bar{\dot{p}} - x}{\bar{\dot{p}} - \bar{\dot{n}}}, & \text{for } \bar{\dot{n}} < x \le \bar{\dot{p}} \\ 0, & \text{for } otherwise \end{array} \right\}$$

$$\widetilde{I}^U = \left\{ \begin{array}{ll} \dfrac{\bar{\dot{r}} - x + \widetilde{I}^U(x - \bar{\dot{q}})}{\bar{\dot{r}} - \bar{\dot{q}}} & \text{for } \bar{\dot{q}} \le x < \bar{\dot{r}} \\ \widetilde{I}^U, & \text{for } \bar{\dot{r}} \le x \le \bar{\dot{s}} \\ \dfrac{x - \bar{\dot{s}} + \widetilde{I}^U(\bar{\dot{t}} - x)}{\bar{\dot{t}} - \bar{\dot{s}}}, & \text{for } \bar{\dot{s}} < x \le \bar{\dot{t}} \\ 1, & \text{for } otherwise \end{array} \right\}$$

$$\widetilde{F}^U = \left\{ \begin{array}{ll} \dfrac{\bar{\dot{v}} - x + \widetilde{I}^U(x - \bar{\dot{u}})}{\bar{\dot{v}} - \bar{\dot{u}}} & \text{for } \bar{\dot{u}} \le x < \bar{\dot{v}} \\ \widetilde{F}^U, & \text{for } \bar{\dot{v}} \le x \le \bar{\dot{w}} \\ \dfrac{x - \bar{\dot{w}} + \widetilde{I}^U(\bar{\dot{x}} - x))}{\bar{\dot{x}} - \bar{\dot{w}}}, & \text{for } \bar{\dot{w}} < x \le \bar{\dot{x}} \\ 1, & \text{for } otherwise \end{array} \right\}$$

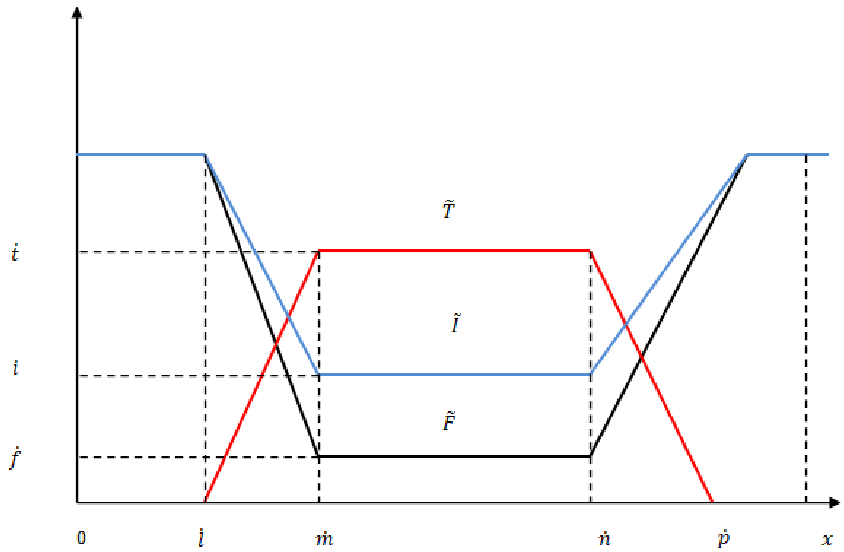

Figure 1: A trapezoidal neutrosophic number

Definition 2.5. Let $\dot{N}^L$ be a lower trapezoidal neutrosophic fuzzy number then, we have these following three trapezoidal neutrosophic fuzzy numbers $\widetilde{T}^L = (\underline{\dot{l}}, \underline{\dot{m}}, \underline{\dot{n}}, \underline{\dot{p}}) : X \to [0,1]$, $\widetilde{I}^L = (\underline{\dot{q}}, \underline{\dot{r}}, \underline{\dot{s}}, \underline{\dot{t}}) : X \to [0,1]$ and $\widetilde{T}^L = (\underline{\dot{u}}, \underline{\dot{v}}, \underline{\dot{w}}, \underline{\dot{x}}) : X \to [0,1]$. If $\underline{\dot{m}} = \underline{\dot{n}}$, $\underline{\dot{r}} = \underline{\dot{s}}$ and $\underline{\dot{v}} = \underline{\dot{w}}$. These trapezoidal neutrosophic fuzzy numbers are reduced to a triangular neutrosophic fuzzy number defined as

$$\widetilde{T}^L = \left\{ \begin{array}{ll} \dfrac{x-\underline{\dot{l}}}{\underline{\dot{m}}-\underline{\dot{l}}} T^L & \text{for} \underline{\dot{l}} \leq x < \underline{\dot{m}} \\ \widetilde{T}^L, & \text{for} \underline{\dot{m}} \leq x \leq \underline{\dot{n}} \\ \dfrac{\underline{\dot{p}}-x}{\underline{\dot{p}}-\underline{\dot{n}}}, & \text{for} \underline{\dot{n}} < x \leq \underline{\dot{p}} \\ 0, & \text{for} \textit{otherwise} \end{array} \right\}$$

$$\widetilde{I}^L = \left\{ \begin{array}{ll} \dfrac{\underline{\dot{r}} - x + \widetilde{I}^L(x-\underline{\dot{q}})}{\underline{\dot{r}}-\underline{\dot{q}}} & \text{for } \underline{\dot{q}} \leq x < \underline{\dot{r}} \\ \widetilde{I}^L, & \text{for } \underline{\dot{r}} \leq x \leq \underline{\dot{s}} \\ \dfrac{x - \underline{\dot{s}} + \widetilde{I}^L(\underline{\dot{t}}-x)}{\underline{\dot{t}}-\underline{\dot{s}}}, & \text{for } \underline{\dot{s}} < x \leq \underline{\dot{t}} \\ 1, & \text{for } \textit{otherwise} \end{array} \right\}$$

$$\widetilde{F}^L = \begin{Bmatrix} \dfrac{\underline{\dot{v}} - x + \widetilde{I}^L(x - \underline{\dot{u}})}{\underline{\dot{v}} - \underline{\dot{u}}} & \text{for } \underline{\dot{u}} \leq x < \underline{\dot{v}} \\ \widetilde{F}^L, & \text{for } \underline{\dot{v}} \leq x \leq \underline{\dot{w}} \\ \dfrac{x - \underline{\dot{w}} + \widetilde{I}^L(\underline{\dot{x}} - x)}{\underline{\dot{x}} - \underline{\dot{w}}}, & \text{for } \underline{\dot{w}} < x \leq \underline{\dot{x}} \\ 1, & \text{for } otherwise \end{Bmatrix}$$

Definition 2.6. [32] Let $n_1 = (\bar{\dot{l}}, \overline{m}, \bar{n}, \overline{p} : \widetilde{T}^U, \widetilde{I}^U, \widetilde{F}^U)$ and $n_2 = (\underline{\dot{l}}, \underline{\dot{m}}, \underline{\dot{n}}, \underline{\dot{p}} : \widetilde{T}^L, \widetilde{I}^L, \widetilde{F}^L)$ are the two IVTrNFN. Therefore, the corresponding Hamming distance between n_1 and n_2 could be reasonably defined by

$$\widetilde{D}(n_1, n_2) = \frac{1}{12}[|\bar{\dot{l}}_1(2+\overline{\tau}-\overline{\iota}-\overline{F}) - \underline{\dot{l}}_2(2+\overline{\tau}-\overline{\iota}-\overline{F})| + |\overline{m}_1(2+\overline{\tau}-\overline{\iota}-\overline{F}) - \underline{\dot{m}}_2(2+\overline{\tau}-\overline{\iota}-\overline{F})| \\ + |\bar{\dot{n}}_1(2+\overline{\tau}-\overline{\iota}-\overline{F} - \underline{\dot{n}}_1(2+\overline{\tau}-\overline{\iota}-\overline{F})| + |\bar{\dot{p}}_1(2+\overline{\tau}-\overline{\iota}-\overline{F}) - \underline{\dot{p}}_2(2+\overline{\tau}-\overline{\iota}-\overline{F})|] \quad (1)$$

Definition 2.7. The expected value I_n of an IVTrNFN can be defined as $n = ([\bar{\dot{l}}, \overline{\dot{m}}, \bar{\dot{n}}, \bar{\dot{p}}], [\underline{\dot{l}}, \underline{\dot{m}}, \underline{\dot{n}}, \underline{\dot{p}}])$ $(\widetilde{T}(x), \widetilde{I}(x), \widetilde{F}(x))$ and is shown as follows

$$I_n = \frac{1}{12}[|\bar{\dot{l}} - \underline{\dot{l}}| + |\overline{\dot{m}} - \underline{\dot{m}}| + |\bar{\dot{n}} - \underline{\dot{n}}| + |\bar{\dot{p}} - \underline{\dot{p}}|] \times [2 + |\widetilde{T}_U - \widetilde{T}_L| - |\widetilde{I}_U - \widetilde{I}_L| + |\widetilde{F}_U - \widetilde{F}_L|] \quad (2)$$

Definition 2.8. Let n_1 and n_2 be two IVTrNFNs

$$n_1 = (\bar{l}_1, \overline{m}_1, \overline{n}_1, \overline{p}_1), (\underline{l}_1, \underline{m}_1, \underline{n}_1, \underline{p}_1 : T_1), (\overline{q}_1, \overline{r}_1, \overline{s}_1, \overline{t}_1), (\underline{q}_1, \underline{r}_1, \underline{s}_1, \underline{t}_1 : I_1) \\ (\overline{u}_1, \overline{v}_1, \overline{x}_1, \overline{z}_1), (\underline{u}_1, \underline{v}_1, \underline{x}_1, \underline{z}_1 : F_1) \quad (3)$$

$$n_2 = (\bar{l}_2, \overline{m}_2, \overline{n}_2, \overline{p}_2), (\underline{l}_2, \underline{m}_2, \underline{c}_2, \underline{p}_2 : T_2)(\overline{q}_2, \overline{r}_2, \overline{s}_2, \overline{t}_2), (\underline{q}_2, \underline{r}_2, \underline{g}_2, \underline{t}_2 : I_2), \\ (\overline{u}_2, \overline{v}_2, \overline{w}_2, \overline{x}_2), (\underline{u}_2, \underline{v}_2, \underline{w}_2, \underline{x}_2 : F_2) \quad (4)$$

Definition 2.9. [36, 35, 48] Therefore, the basic operations on IVTrNFNs could be defined as follows

- **addition**
  $n_1 \oplus n_2 = \langle \Big\{ (\underline{l_1} + \underline{l_2} - \underline{l_1 l_2}, \underline{m_1} + \underline{m_2} - \underline{m_1 m_2}, \underline{n_1} + \underline{n_2} - \underline{n_1 n_2}, \underline{p_1} + \underline{p_2} - \underline{p_1 p_2}),\ (\overline{l_1} + \overline{l_2} - \overline{l_1 l_2}, \overline{m_1} + \overline{m_2} - \overline{m_1 m_2}, \overline{n_1} + \overline{n_2} - \overline{n_1 n_2}, \overline{p_1} + \overline{p_2} - \overline{p_1 p_2}) \Big\}, \Big\{ (\underline{q_1 q_2}, \underline{q_1 q_2}, \underline{r_1 r_2}, \underline{r_1 r_2}), (\overline{s_1 s_2}, \overline{s_1 s_2}, \overline{t_1 t_2}, \overline{t_1 t_2}),\ (\underline{u_1 u_2}, \underline{u_1 u_2}, \underline{v_1 v_2}, \underline{v_1 v_2}),\ (\overline{w_1 w_2}, \overline{w_1 w_2}, \overline{x_1 x_2}, \overline{x_1 x_2}) \Big\}$

- **Multiplication**
  $n_1 \otimes n_2 = [(\underline{l}_1 \underline{l}_2, \underline{m}_1 \underline{m}_2, \underline{n}_1 \underline{n}_2, \underline{p}_1 \underline{p}_2), (\bar{l}_1 \bar{l}_2, \overline{m}_1 \overline{m}_2, \overline{n}_1 \overline{n}_2, \overline{p}_1 \overline{p}_2)]$

$[(\underline{q}_1+\underline{q}_2-\underline{q}_1\underline{q}_2,\underline{r}_1+\underline{r}_2-\underline{r}_1\underline{r}_2,\underline{s}_1+\underline{s}_2-\underline{s}_1\underline{s}_2,\underline{t}_1+\underline{t}_2-\underline{t}_1\underline{t}_2)]$
$[(\underline{u}_1+\underline{u}_2-\underline{u}_1\underline{u}_2,\underline{v}_1+\underline{v}_2-\underline{v}_1\underline{v}_2,\underline{w}_1+\underline{w}_2-\underline{w}_1\underline{w}_2,\underline{x}_1+\underline{x}_2-\underline{x}_1\underline{x}_2)][$
$[(\overline{u}_1+\overline{u}_2-\overline{u}_1\overline{u}_2,\overline{v}_1+\overline{v}_2-\overline{v}_1\overline{v}_2]$
$\overline{w}_1+\overline{w}_2-\overline{w}_1\overline{w}_2,\overline{x}_1+\overline{x}_2-\overline{x}_1\overline{x}_2)]$

- **Scalar Multiplication**

$$\lambda n_1=\langle\Big\{(1-(1-\underline{l}_1)^\lambda,1-(1-\underline{m}_1)^\lambda$$

$$,1-(1-\underline{n}_1)^\lambda,1-(1-\underline{p}_1)^\lambda),(1-(1-\bar{l}_1)^\lambda,1-(1-\overline{m}_1)^\lambda,1-(1-\overline{n}_1)^\lambda,1-(1-\overline{p}_1)^\lambda)\Big\},$$

$$[(\underline{q}_1^\lambda,\underline{r}_1^\lambda,\underline{s}_1^\lambda,\underline{t}_1^\lambda),(\overline{q}_1^\lambda,\overline{r}_1^\lambda,\overline{s}_1^\lambda,\bar{t}_1^\lambda)][(\underline{u}_1^\lambda,\underline{v}_1^\lambda,\underline{w}_1^\lambda,\underline{x}_1^\lambda),(\overline{u}_1^\lambda,\overline{v}_1^\lambda,\overline{w}_1^\lambda,\overline{x}_1^\lambda)]\rangle \quad (5)$$

- **Exponentiation**

$n_1^\lambda= [(\underline{l}_1^\lambda,\underline{m}_1^\lambda,\underline{n}_1^\lambda,\underline{p}_1^\lambda),(\bar{l}_1^\lambda,\overline{m}_1^\lambda,\overline{n}_1^\lambda,\overline{p}_1^\lambda)],\Big\{(1-(1-\underline{q}_1)^\lambda,1-(1-\underline{r}_1)^\lambda,1-(1-\underline{s}_1^\lambda),1-(1-$
$\underline{t}_1\lambda)),(1-(1-\overline{q}_1)^\lambda,1-(1-\overline{r}_1)^\lambda,1-(1-\overline{s}_1^\lambda),1-(1-\bar{t}_1\lambda))\Big\}\Big\{(1-(1-\underline{u}_1)^\lambda,1-(1-$
$\underline{v}_1)^\lambda,1-(1-\underline{w}_1^\lambda),1-(1-\underline{x}_1\lambda))(1-(1-\overline{u}_1)^\lambda,1-(1-\overline{v}_1)^\lambda,1-(1-\overline{w}_1^\lambda),1-(1-\overline{x}_1\lambda))\Big\}$

3 Multi-Criteria Decision Making (MCDM) strategy based upon the Interval-valued Trapezoidal neutrosophic numbers

Regarding the multi-attribute decision making technique (MADMT) problem m alternatives are considered $A=(A_1,A_2,.....A_m)$, n decision criteria $Q=Q_1,Q_2....Q_n$ and $\omega=(\omega_1,\omega_2,........\omega_n)$ is the corresponding weight vector of an attribute with $\omega_j\in[0,1]$ and their specific valuesw_j not defined. The attribute number of an alternative A_i on the criteria Q_j is the interval-valued trapezoidal neutrosophic fuzzy number $n_{ij}=([a_{ij}^1,a_{ij}^2,a_{ij}^3,a_{ij}^4],[\tau(x),\iota(x),F(x)]$ where, the $\tau(x)$ tells us about the extent the alternative A_j belongs to IVTrNFN $[a_{ij}^1,a_{ij}^2,a_{ij}^3,a_{ij}^4]$ and the $\iota(x) and F(x)$ tell us about the extent the alternative A_j does not belong to IVTrNFN $[a_{ij}^1,a_{ij}^2,a_{ij}^3,a_{ij}^4]$ on the criteria Q_j $0\le\tau(x)+\iota(x)+F(x)\le1$ and the decision matrix (DM) denoted $D=[a_{ij}]$ constructs the Positioning of alternative required.
The sequence of computation steps in the Decision Making (DM) methodology of the (IVTrNFN) are defined as follows:

3.1 Strategy and the algorithm to standardize the Deciision Making (DM) process

To eliminate the difference in the scaling of various physical dimensions in the computations defined in the DM process, at first normalization is performed. Suppose $N = ([n_{ij}]_{m\times n})$ denotes the decision matrix where $n^z_{ij} = [n^1_{ij}, n^2_{ij}, n^3_{ij}, n^4_{ij}]$, then, the methodology for normalization of two distinct kind of criteria is given below :

3.2 Cost type criteria

$$n^p_{ij} = \frac{max(a^1_{ij}) - a^p_{ij}}{max(a^4_{ij}) - min(a^1_{ij})} \quad z = 1, 2, 3, 4 \tag{6}$$

3.3 Benefit type criteria

$$n^p_{ij} = \frac{a^p_{ij} - min(a^1_{ij})}{max(a^4_{ij}) - min(a^1_{ij})} \quad z = 1, 2, 3, 4 \tag{7}$$

3.4 Calculating the attribute weight

There are many techniques in the literature to calculate the attributes weights in Decision Making. In the herein research the information entropy technique is employed and discussed as the most suitable and computationally effective for managing uncertainty in weights calculation. Entropy is an important concept of the thermodynamics originated by Shannon [3] who has, also, introduced the method of the entropy as equivalent to the uncertainty. Entropy is a concept used in all disciplines ranging from management science to engineering science. Entropy involves finding the measurement of disorder, unevenness of distribution etc. in physical parameters. In mathematics entropy is involved to find the uncertainty and the quality of effective information.

Entropy is a concept easily understandable and related to fuzziness, so that the highest the fuzziness characterizes a system, then, the greatest its entropy value (EP) would be. Therefore, we know that if the attribute values of all alternatives have small differences regarding the specific attribute then, it is associated with the highest EP. It provides adequate knowledge to define the position of a specific alternative, by assuming it possesses a little value in the prioritizing technique. On the other hand, if the attribute values of all alternatives have obvious differences then, this attribute presents little entropy and similarly is helping to determine the best alternative. So, in order to figure out the alternatives, if one attribute has the highest entropy it is assigned with the lowest weight and if it has the lowest entropy then, it is assigned with the highest weight.

3.5 A Technique to calculate the entropy value of attribute

To verify the entropy value of an attribute the formulae given below are involved:

$$E_j = -Z \times \sum_{x=1}^{n} f_{ij} ln f_{xy} \;\; (1 \leq i \leq p, 1 \leq j \leq q) \tag{8}$$

where $= Z \frac{1}{lpq}$ and if $f_{ij} = 0$, then, $0 \times ln0 = 0$

$$f_{ij} = \frac{I(r_{ij})}{\sum_{x=1}^{q} I_{(}r_{ij})} \;\; (1 \leq i \leq p), (1 \leq j \leq q) \tag{9}$$

The entropy weight is calculated as:

$$\omega_j = \frac{(1 - H_j)}{\sum_{x=1}^{q} (1 - H_j)} \tag{10}$$

The equation mentioned above is used to find the weight. But a research effort. namely, that of Zhou et al[29]. comes with the refined formula for measuring the weight as follows :

$$w_j = \frac{\sum_{j=1}^{n} H_j + 1 - 2 \times H_j}{\sum_{j=1}^{q} \left(\sum_{j=1}^{q} H_y + 1 - 2 \times H_j \right)} \; (1 \leq y \leq q) \tag{11}$$

3.6 Positioning the alternatives based on Grey Relational Projection Technique (GRPT)

Deng [25, 26] introduced and discussed the concepts of the Grey Relational theory and its computational method dealing with the system scrutiny under incomplete knowledge. Grey Relational Analysis recommended the uncertain relation among things, complex systems of elements and their behaviors. Grey Relational theory is the process of quantitative analysis scrutiny of alternatives. Grey Relational theory is used in many real world applications spanning recruitment and hiring processes, employment, power distribution systems operation and decision making processes.

Multi-Attribute Decision Making (MADM) problems can be solved by using the GRA approach. Concerning the application of this methodology the entire domain of attribute values of the system under consideration is combined for every alternative into one only value. The result of such an operation is the reduction of the original problem to a single attribute decision making problem. Therefore, alternative decisions with multiple attributes can be compared easily after the application of the above specified GRA process. The process of combining attribute values into a single value is similar to the method adopted in SAW and TOPSIS[44].

The main procedure of GRPT is converting the attribute values of all alternatives into a comparability sequence by reducing the effects from different physical parameters. Based on these sequences, the methodology determines a reference sequence, the so called ideal target sequence or negative ideal target sequence. Then, this approach calculates the grey relational coefficient between all such comparability sequences and the reference sequence. Finally, based on these grey relational coefficients, the grey relational degree between the reference sequence and every such considered comparability sequence is completely determined. If a comparability sequence that is converted from an alternative decision has the highest grey relational degree with the reference sequence, in case it is the ideal target sequence, or the lowest grey relational degree with the reference sequence, in case it is the negative ideal target sequence, that alternative solution will be the best choice. Suppose then, the decision making matrix (DM) has been normalized as previously outlined, the steps of grey relational projection technique are illustrated as follows:

3.7 Optimistic ideal solution (OIS) and Pessimistic ideal solution (PIS)

If a normalized IVTrNFN decision matrix has been determined, then, an IVTrNFN Optimistic ideal solution(OIS) and the corresponding IVTrNFN Pessimistic ideal solution (PIS) could be defined as follows. Regarding the OIS definition:

$$n^{+} = \left(n_1^{+}, n_2^{+}, ..., n_p^{+}\right) \tag{12}$$

where n_p^{+} is evaluated as:

$$\begin{aligned} n_j^{+} &= \left([n_j^1+, n_j^2+, n_j^3+, n_j^4+]\right); T(N_x), I(N_x), F(N_x) \\ &= \left([\underbrace{\max_i}(n_{ij}^1), \underbrace{\max_i}(n_{ij}^2), \underbrace{max_i}(n_{ij}^3), \underbrace{\max_i}(n_{ij}^4)]; \underbrace{\max_i} \tau(N_x), \underbrace{\min_i} \iota(N_x), \underbrace{\min_i} F(N_x) \right) \end{aligned} \tag{13}$$

Moreover, the Fuzzy trapezoidal Pessimistic Ideal Solution (PIS) can be evaluated using:

$$n^{-} = \left(n_1^{-}, n_2^{-}, ..., n_q^{-}\right) \tag{14}$$

where, each of n_q^- can be calculated using the following relation,

$$n_j^- = \left([n_j^1-, n_j^2-, n_j^3-, n_j^4-]\right); T(N_x), I(N_x), F(N_x)$$
$$= \left([\underbrace{\max_i}(n_{ij}^1), \underbrace{\max_i}(n_{ij}^2), \underbrace{\max_i}(n_{ij}^3), \underbrace{\max_i}(n_{ij}^4)]; \underbrace{\min_i} \tau(N_x) \underbrace{\max_i}, \iota(N_x) \underbrace{\max_i}, F(N_x)\right) \tag{15}$$

Based on the above and similarly to [27], another direct definition of the corresponding Fuzzy optimistic ideal solution (OIS) could be:

$n_j^+ = ([1, 1, 1, 1]; 1, 1, 1)$ and the relevant definition of the Fuzzy Pessimistic Ideal Solution (PIS) is $n_j^- = ([0, 0, 0, 0]; 0, 0, 0)$

3.8 Calculation of the Grey relational coefficient (GRC)

The GRC of each alternative from (OIS) and (PIS) can be calculated using the following equations respectively. The GRC of each alternative from (OIS) is given as :

$$\varsigma_{ij}^+ = \frac{S^+ + \varrho E^+}{d_{ij}^+ + \varrho P^+} \tag{16}$$

where, the Normalized hamming distance

$$D(n_1, n_2^+) = \frac{1}{12}[|(2 + \overline{\tau} - \overline{\iota} - \overline{F})n_{ij}^1 - (2 + \overline{\tau} - \overline{\iota} - \overline{F})n_j^1 + |+$$
$$|(2+\overline{\tau}-\overline{\iota}-\overline{F})n_{ij}^2-(2+\overline{\tau}-\overline{\iota}-\overline{F})n_j^2+|+|(2+\overline{\tau}-\overline{\iota}-\overline{F})n_{ij}^3-(2+\overline{\tau}-\overline{\iota}-\overline{F})n_j^3+|+|(2+\overline{\tau}-\overline{\iota}-\overline{F})n_{ij}^4-$$
$$(2 + \overline{\tau} - \overline{\iota} - \overline{F})n_{ij}^4 + | \tag{17}$$

$$S^+ = \underbrace{\min_i} \underbrace{\min_j} d_{ij}^+, P^- = \underbrace{\max_i} \underbrace{\max_j} d_{ij}^+ \tag{18}$$

, ϱ is the resolution coefficient and $\varrho \varepsilon (0, 1)$ and its value could be considered 0.5 in case of incomplete and uncertain knowledge
So, the GRC of each alternative in (OIS) is given as

$$\varsigma_{ij}^- = \frac{S^- + \varrho E^-}{d_{ij}^- + \varrho P^-} \tag{19}$$

where, the normalized hamming distance is

$$D(n_1, n_2^-) = \frac{1}{12}[|(2 + \overline{\tau} - \overline{\iota} - \overline{F})n_{ij}^1 - (2 + \overline{\tau} - \overline{\iota} - \overline{F})n_j^1 - |+$$
$$|(2+\overline{\tau}-\overline{\iota}-\overline{F})n_{ij}^2-(2+\overline{\tau}-\overline{\iota}-\overline{F})n_j^2-|+|(2+\overline{\tau}-\overline{\iota}-\overline{F})n_{ij}^3-(2+\overline{\tau}-\overline{\iota}-\overline{F})n_j^3-|+|(2+\overline{\tau}-\overline{\iota}-\overline{F})n_{ij}^4-$$
$$(2 + \overline{\tau} - \overline{\iota} - \overline{F})n_{ij}^4 - | \tag{20}$$

.

$$S^{-} = \underbrace{\min_{i}}\underbrace{\min_{j}} d_{ij}^{-}, P^{-} = \underbrace{\max_{i}}\underbrace{\max_{j}} d_{ij}^{-} \tag{21}$$

ϱ is the resolution coefficient and $\varrho\varepsilon(0,1)$ and its value is 0.5 again in case of complete ignorance and uncertainty
Similarly, the GRC of each alternative in (PIS) is given as

$$\varsigma_{ij}^{-} = \frac{S^{-} + \varrho E^{-}}{d_{ij}^{-} + \varrho P^{-}} \tag{22}$$

3.9 Calculation of Grey relational grade GRG

We can find the GRG of every alternative from (OIS) and (PIS) by using the formulae given below.

$$\varsigma_{i}^{+} = \sum_{j=1}^{q} w_j \varsigma ij^{+} \tag{23}$$

$$\varsigma_{i}^{-} = \sum_{j=1}^{q} w_j \varsigma ij^{-} \tag{24}$$

3.10 Positioning the alternative

The positioning process in the GRA method is the procedure of selecting the alternative with the "greatest degree of grey relation" from (OIS) and the "smallest degree of grey relation" from (PIS).

3.11 Projection

The weight associated with the GPC (Grey relational Projection coefficient) will be

$$\overline{w}_j = \frac{w_j^2}{\sqrt{\sum_{j=1}^{q} w_j^2}} \tag{25}$$

In the sequel the weighted grey correlation projection technique is applied for the alternative from the (OIS) solution and the (PIS) as follows:

$$\wp_i^{+} = \sum_{j}^{q} (\overline{w}_j \times \varsigma_{ij}^{+}) \tag{26}$$

$$\wp_i^{-} = \sum_{j}^{q} (\overline{w}_j \times \varsigma_{ij}^{-}) \tag{27}$$

3.12 Similarity approximation to OIS and Positioning an alternative

The alternative under consideration can now be positioned based on the GPC projection coefficient of each and every alternative on the basis of the OIS and the PIS solutions. The greatest the projected number on the OIS solution is, then, the closest to OIS and the best the alternative would be, while on the other hand, the smallest the projection onto PIS is, the more away the alternative to PIS should be and the better the considered alternative would be. Therefore, the similarity closeness can be defined as

$$RC_i = \frac{\sum_j^q (\overline{w}_j \times \varsigma_{ij}^+)}{\sum_j^q (\overline{w}_j + \wp_i^-) \sum_j^q (\overline{w}_j \times \varsigma_{ij}^-)} \tag{28}$$

4 An Illustrative Numerical Example for the Application of the proposed GRPT methodology based on IVTrNFN representation framework

From the beginning of the mankind a multitude of different viral diseases have been arose with the passage of time. In the herein numerical example three viral diseases with common symptoms are discussed and the responsible Medical staff should be able to efficiently make a decision about the viral disease under consideration on the basis of the symptoms provided. Let A_1, A_2, A_3 are the three viral diseases named as **Spanish flu** , **Covid-19** and **Swine Flu** having the following four different symptoms Q_1, Q_2, Q_3, Q_4 given below .

- Abdominal pain , lethargy, sneezing , weight loss.
- High fever,tiredness, dry cough ,Breathing issues Fever.
- Cough , sorethroat, joint pain , body aches.
- Earache , Petechial rashes, Hoarseness, Nasal congestion.

The assumption of the alternatives A_1, A_2, A_3 would be to be represented in the form of IVTrNFN w.r.t four attributes Q_1, Q_2, Q_3, Q_4. The Decision Matrix (DM) will be in the form

Table 1: Attributive values of alternatives

Alternative⇒	Q_1	Q_2	Q_3	Q_4
A_1	$(1,2,3,4)(0.1,0.2,0.4)$ $(2,4,8,9)(0.2,0.3,0.5)$	$(2,5,7,8)(0.2,0.3,0.7)$ $(3,4,5,10)(0.4,0.5,0.6)$	$(3,7,8,9)(0.3,0.4,0.8)$ $(7,8,9,10)(0.5,0.6,0.9)$	$(1,5,7,9)(0.1,0.3,0.2)$ $(2,4,5,7)(0.4,0.5,0.9)$
A_2	$(1,5,6,9)(0.1,0.3,0.5)$ $(2,3,5,7)(0.2,0.5,0.8)$	$(1,3,7,8)(0.1,0.2,0.6)$ $(1,2,7,5)(0.5,0.3,0,7)$	$(1,2,8,9)(0.1,0.2,0.3))$ $(1,3,5,8)(0.4,0.7,0.9)$	$(5,6,8,9)(0.3,0.4,0.5)$ $(2,5,8,6)(0.5,0.6,0.9)$
A_3	$(3,6,4,9)(0.1,0.2,0.4)$ $(1,2,4,7)(0.3,0.6,0.7)$	$(1,5,8,9)(0.1,0.3,0.5)$ $(2,5,8,10)(0.5,0.2,0.9)$	$(4,5,6,7)(0.7,0.8,0.9)$ $(1,2,8,9)(0.8,0.9,o.7)$	$(1,3,5,7)(0.3,0.5,0.7)$ $(2,5,7,9)(0.4,0.8,0.9)$

The computational steps of the application of the herein proposed methodology for the problem at hand are as follows:

Step I.

Standardize the matrix

$N = (\tilde{n}^{ij})_{m\times n}$ using the equation 6 and 7

Step II.

The notion of expected value arises and it could be defined as the central value of the corresponding interval. The expected value is calculated from the entropy weight as previously analyzed and could be defined by using the equation2 and 1.

$$I_n = \begin{bmatrix} 2.05 & 0.9 & 1.108 & 0.7 \\ 0.8 & 0.733 & 0.5833 & 0.933 \\ 1 & 0.316 & 1.5 & 0.933 \end{bmatrix}$$

The attribute weights could be calculated as

$\omega_1 = 0.247$ $\omega_2 = 0.433$,$\omega_3 = 0.3193$

Step III.

As it has been already analyzed the optimistic ideal solution (OIS) maximizes the benefits criterion and minimizes the cost criterion. On the other hand, the pessimistic ideal solution (PIS) minimizes the benefits criterion and maximizes the cost criterion of the IVTrNFN. Such ideal alternatives could be verified with the help of the equations 16 and 19

Table 2: Standardize values of alternatives

⇒	Q_1	Q_2	Q_3	Q_4
A_1	$(0,0.12,0.2,0.37)(0.1,0.2,0.4)$ $(0,0.25,0.75,0.87)(0.2,0.3,0.5)$	$(0.2,0.6,0.7,0.8)(0.2,0.3,0.7)$ $(0.6,0.7,0.8,1)(0.4,0.5,0.6)$	$(0,0.5,0.6,1)(0.3,0.4,0.8)$ $(0.12,0.25,0.5,0.75)(0.5,0.6,0.9)$	$(0,0.2,0.6,0.7)(0.1,0.3,0.2)$ $(0,0.1,0.6,0.4)(0.4,0.5,0.9)$
A_2	$(0,0.12,0.8,1)(0.1,0.3,0.5)$ $(0,0.25,0.5,0.8)(0.2,0.5,0.8)$	$(0.5,0.62,0.87,1)(0.1,0.2,0.6)$ $(0.12,0.5,0.87,0.6)(0.5,0.3,0,7)$	$(0.5,0.62,0.87,1)(0.1,0.2,0.3))$ $(0.12,0.5,0.87,0.6)(0.4,0.7,0.9)$	$(0.25,0.62,0.37,1)(0.3,0.4,0.5)$ $(0,0.12,0.37,0.75)(0.5,0.6,0.9)$
A_3	$(0.25,0.62,0.37,1)(0.1,0.2,0.4)$ $(0,0.12,0.37,0.75)(0.3,0.6,0.7)$	$(0,0.44,0.7,0.8)(0.1,0.3,0.5)$ $(0.1,0.4,0.7,1)(0.5,0.2,0.9)$	$(0.37,0.5,0.62,0.75)(0.7,0.8,0.9)$ $(0,0.12,0.87,0.17)(0.8,0.9,0.7)$	$(0,0.25,0.5,0.75)(0.3,0.5,0.7)$ $(0.125,0.5,0.75,1)(0.4,0.8,0.9)$

$$\varsigma_j^+ = \{([0.25, 0.62, 0.75, 0.87](0.3, 0.2, 0.4)[0.6, 0.62, 0.87, 0.87](0.5, 0.2, 0.5) \\ [0.37, 0.62, 0.87, 0.75](0.8, 0.2, 0.3)[0.25, 0.62, 0.75, 0.75](0.5, 0.3, 0.2))\} \tag{29}$$

$$\varsigma_j^- = \begin{array}{c}\{([0, 0.12, 0.2, 0.37](0.1, 0.6, 0.8)[0, 0.4, 0.7, 0.6](0.1, 0.5, 0..9)[0, 0.25, 0.5, 0.67] \\ (0.1, 0.9, 0.9)[0, 0.25, 0.37, 0.4](0.1, 0.8, 0.9))\}\end{array} \tag{30}$$

Step IV. GRC is verified with respect to the relationship between the ideal and the actual values. The GRC for the OIS optimistic solution and the PIS, the negative ideal solution, are herein verified with the help of equations 23 and 24

$$(\varsigma_{ij}^+)_{3\times4} = \begin{bmatrix} 0.668 & 0.7186 & 0.619 & 0.6234 \\ 0.958 & 1.000 & 0.661 & 0.5965 \\ 0.6753 & 0.707 & 0.8033 & 0.788 \end{bmatrix}$$

$$(\varsigma_{ij}^-)_{3\times4} = \begin{bmatrix} 0.810 & 0.8961 & 0.610 & 0.7909 \\ 0.893 & 1.000 & 0.824 & 0.752 \\ 0.827 & 0.8591 & 0.956 & 0.952 \end{bmatrix}$$

Step V.
After calculating the GRC the grey weighted correlation projection coefficient (GWCP) is calculated for every alternative A_i on the OIS and PIS solutions by using equations 26 and 27:

$P_1^+ = 0.23866$, $P_2^+ = 0.3133$, $P_3^+ = 0.25573$
$P_1^- = 0.27966$, $P_2^- = 0.7948$, $P_3^- = 0.30904$

Step VI.

Then, the relative closeness is calculated by using 28:
$RC_1 = 0.46045$, $RC_2 = 0.28144$, $RC_3 = 0.45280$

Step VII.
Finally, the process of positioning the alternative according to its similarity approximation

to each OIS solution is taking place

$$N_2 > N_3 > N_1 \tag{31}$$

So herein the three different types of Flu with their common symptoms are ordered and explained according to the proposed generalized GRA methodology. The proposed framework provides, therefore, a tool helpful for the Viral diagnostic team in the diagnosis of the Flu type.

5 Comparative analysis

A comparative qualitative methodological discussion is developed to show the effectiveness of the proposed approach compared with previous techniques. GRA is appropriate for solving problems with complicated interrelationships amid multiple factors and variables. GRA tactic is employed to examine the uncertainties in the organism and the correlation among the relevant systems and subsystems etc. It is a mathematically oriented hypothesis derived from the idea of the grey set. It is one of the best techniques to solve uncertainty setbacks with partial information and discrete data. In all the previous researches on the GRA most of the researches evolve the concepts of the IFS and IVFS which do not provide adequate information about the indeterminate and the inconsistent character that exist in every real situation. The IVTrNFN representation framework herein employed to enrich classical GRA provides the additional information about the uncertain ,imprecise,incomplete and inconsistent information that exist in real life decision making. IVTrNFN is more suitable to be utilized in the indeterminate and inconsistent environment of complex Decision Making problems including Business Decision Making and Medical Decision Making. In the IFS representation the fuzzy set is defined w.r.t the Truth (T) and therefore, the associated membership is single value only with the information about the indeterminate and inconsistent nature of decision making processes to be nullified. Similarly, in the IVFS representation framework of fuzzy sets these are defined w.r.t to the membership and the nonmembership only. While in the IVTrNFN representation framework the fuzzy sets involved in the GRA application are defined taking into account all aspects of information on MCDM and MADM problems w.r.t to the (T, I, F) vector valued membership. Such an approach is more general and much more efficient to explain and involve the ambiguity, uncertainty and fuzziness arising in the information management of complex DM processes and integrate them naturally in the GRA solution framework related to such complex MCDM and MADM problems.

6 Conclusion and prospects

In all the previous researches regarding the application of the Grey Relational Analysis (GRA) methodology most of the researches evolve and involve the representation framework of the IFS and the ITFS theory which does not provide adequate information about the indeterminate and the inconsistent character of the alternative strategies inherent in real world complex Multi-Criteria (MCDM) and Multi-Attribute decision making (MADM) problems that exist in every real life situation. The IVTrNFN representation framework provides the required additional information for successfully solving such problems that is, suitable measures of uncertainty ,impreciseness,incompleteness and inconsistency regarding the information involved in such decision making processes. The IVTrNFN framework is much more suitable to be utilized in defining the indeterminate and inconsistent parameters in Decision Making Techniques (DMT). In the IFS representation the fuzzy set is defined w.r.t to Truth (T) as well as its associated memberships. Therefore, the indeterminate and the inconsistent state of information is nullified and not considered. Moreover, even the involvement of the Interval Valued Fuzzy Sets (IVFS) and the Interval Valued Intuitionistic Fuzzy Sets (IVIFS) theory cannot provide all required means to effectively manage uncertainty of information in MCDM and MADM problems, since the parameters using IVFS or IVIFS are defined w.r.t to the membership and the non-membership only, regarding Truth of Information (T). On the other hand, in the IVTrNFN representation the relevant parameters are defined w.r.t to the (T, I, F) vector of Truth, Indeterminacy and Falsity, which constructs a much more general and proper framework to explain the ambiguity, uncertainty and fuzziness that arises in the integration of decision making processes and strategies information. In the herein presented study w.r.t the MADM situation, the proposed framework and methodology has been used to cope up with the attributes values involving the Interval-valued trapezoidal Neutrosophic fuzzy numbers (IVTrNFN) representation with null knowledge about the weight of the attribute. After the introduction of basic arithmetic laws, the calculation of expected values and the corresponding normalized hamming distances between two IVTrNFN numbers are illustrated. In the sequel, the information Entropy Technique is used to estimate the weight value of the MADM attributes in order that the Grey Relational Analysis (GRA) could be developed. In the proposed generalized GRA methodology the alternative decision making strategies have been ranked by introducing the similarity closeness with respect to the Optimistic Ideal Solution (OIS) and merging the GRA values from the OIS and the Pessimistic Ideal Solution (PIS) for each alternative. Finally, we illustrate the application of the proposed methodology in deriving the numerical solution of a Medical Decision Making example in order to explain the detailed concepts involved and to demonstrate its computational applicability and efficiency. Based on this research effort a quantitative comparative study is planned by the authors regarding the applicability, computational efficiency and representational power of the different GRA approaches discussed in solving complex MCDM and MADM problems.

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