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Combined Effects of Viscous Dissipation, Joule Heating and Thermal Radiation on MHD Axisymmetric Flow of Power-Law Fluid over an Unsteady Stretching Sheet

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Abstract

In this study, the unsteady magneto-hydrodynamic (MHD) axisymmetric flow of a power-law fluid over an unsteady stretching sheet is investigated, taking into account the combined effects of viscous dissipation, Joule heating, and thermal radiation. The governing system of nonlinear partial differential equations is transformed into a system of coupled ordinary differential equations using appropriate similarity transformations. The resulting boundary value problem is solved numerically using the shooting method integrated with a fourth-order Runge-Kutta integration scheme. The influences of key dimensionless parameters—namely, the magnetic parameter, power-law index, unsteadiness parameter, thermal radiation parameter, and Biot number—on the velocity and temperature distributions are analyzed in detail. Additionally, the roles of viscous dissipation (via the Eckert number) and Joule heating in modifying the thermal boundary layer are evaluated. The results indicate that an increase in the magnetic parameter leads to a significant decrease in the flow velocity due to the resistive Lorentz force, while viscous dissipation and Joule heating enhance the temperature field. Furthermore, the thermal boundary

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layer thickness increases with rising thermal radiation. Crucial engineering quantities, such as the skin friction coefficient and local Nusselt number, are computed and discussed. This study provides insightful details on the rheological and thermal interactions of non-Newtonian fluids, which are highly relevant to various industrial and materials processing applications.

Keywords: Power-law fluid; Joule heating; Viscous dissipation; Thermal radiation; Stretching sheet; Shooting method

1. Introduction

The study of non-Newtonian fluid flow has received considerable attention owing to its broad range of applications in industrial, engineering, and technological processes. Unlike Newtonian fluids, non-Newtonian fluids exhibit a nonlinear relationship between shear stress and shear rate, making their flow behavior significantly more complex. Such fluids are commonly encountered in polymer solutions, paints, blood, food products, drilling muds, and molten plastics. Among the various rheological models developed to characterize non-Newtonian fluids, the power-law model has gained widespread acceptance due to its simplicity and effectiveness in describing both shear-thinning and shear-thickening behaviors. Consequently, the analysis of power-law fluid flow has become an important topic in fluid mechanics and heat transfer research. Chhabra and Richardson [1] provided a comprehensive treatment of non-Newtonian rheology, while Bird et al**.** [2] established the theoretical foundations of polymeric fluid mechanics. Acrivos et al. [3] further investigated momentum and heat transfer characteristics of non-Newtonian boundary-layer flows.

Boundary-layer flow over stretching surfaces has attracted substantial interest because of its numerous applications in extrusion processes, wire drawing, glass-fiber production, polymer sheet manufacturing, metal spinning, and continuous casting operations. The pioneering work of L. J. Crane [4] initiated extensive investigations into stretching sheet problems and motivated researchers to examine various physical effects influencing momentum and thermal transport. In many practical situations, stretching surfaces operate under time-dependent conditions, making unsteady flow analysis essential for accurately representing industrial processes. Furthermore, axisymmetric flow configurations provide a more realistic description of several engineering systems than two-dimensional models and therefore have received increasing attention in recent years. Vajravelu and Rollins [5] investigated heat transfer in electrically conducting fluids over stretching surfaces, whereas Ellahi et al. [6] examined non-Newtonian flow behavior with slip effects over stretching sheets.

The interaction of electrically conducting fluids with externally applied magnetic fields gives rise to magneto-hydrodynamic (MHD) flows, which play a vital role in many modern engineering applications. MHD flow phenomena are encountered in electromagnetic casting, metallurgical processing, cooling systems of nuclear reactors, MHD generators, crystal growth technologies, plasma confinement devices, and geothermal energy extraction systems. The application of a magnetic field generates a Lorentz force that opposes fluid motion and significantly modifies the velocity and thermal fields. Consequently, MHD flow analysis has become an important research area for improving heat transfer performance and controlling fluid motion in electrically conducting fluids. Davidson [7] and Shercliff [8] established the theoretical foundations of magneto-hydrodynamics. Wang [9] investigated three-dimensional stretching surface flows, while Ahmed et al. [10] extended the analysis to axisymmetric MHD power-law fluid flow under unsteady conditions.

Heat transfer characteristics in non-Newtonian MHD flows are often influenced by several additional physical mechanisms. Among these, viscous dissipation, Joule heating, and thermal radiation are particularly important in high-temperature industrial operations. Viscous dissipation represents the conversion of mechanical energy into thermal energy due to internal fluid friction and becomes significant in highly viscous fluids and high-speed flows. Joule heating arises from the interaction between electrical currents and magnetic fields and contributes additional thermal energy to the fluid system. Thermal radiation, on the other hand, becomes increasingly important at elevated temperatures where radiative heat transfer may dominate conductive and convective modes of heat transport. The combined influence of these mechanisms can substantially alter temperature distributions, thermal boundary-layer thicknesses, and heat transfer rates in practical engineering systems. Bejan [11] discussed convective heat transfer mechanisms in detail. Makinde [12] analyzed viscous dissipation effects, Rashidi et al. [13] studied thermal radiation in MHD flows, Chamkha [14] investigated Joule heating effects, and Hayat et al. [15] reported significant thermal radiation effects in non-Newtonian fluid transport.

Several researchers have investigated MHD flow and heat transfer in non-Newtonian fluids under different physical conditions. Anderson et al. [16] analyzed the flow characteristics of power-law fluids over stretching surfaces and highlighted the influence of rheological parameters on momentum transport. Chen

[17] examined heat transfer behavior in power-law fluids subjected to stretching sheet conditions and demonstrated the importance of non-Newtonian effects on thermal performance. Makinde and co-workers [18] investigated various MHD boundary-layer flow problems involving convective boundary conditions and heat transfer enhancement mechanisms. Other researchers have considered thermal radiation, viscous dissipation, and Joule heating separately in different MHD flow configurations and demonstrated their important contributions to thermal transport phenomena. Chamkha [19] investigated Joule heating effects in MHD flows. Swain et al. [20] analyzed the combined influence of viscous dissipation and Joule heating in MHD transport phenomena.

Despite the considerable amount of literature available on MHD power-law fluid flow, relatively limited attention has been devoted to the simultaneous incorporation of viscous dissipation, Joule heating, and thermal radiation in an unsteady axisymmetric MHD flow of a power-law fluid over a stretching sheet. The combined interaction of these physical mechanisms is highly relevant to modern thermal processing systems and advanced manufacturing technologies, where accurate prediction of flow and heat transfer characteristics is essential. The lack of comprehensive studies addressing these effects simultaneously constitutes an important research gap that motivates the present investigation.

Hence, the primary objective of this investigation is to study the combined effects of viscous dissipation, Joule heating, and thermal radiation on the unsteady axisymmetric MHD flow of a power-law fluid over an unsteady stretching sheet. By deploying suitable similarity transformations, the governing nonlinear partial differential equations are reduced to a system of coupled ordinary differential equations. A numerical solution is subsequently obtained for the boundary value problem using the shooting technique paired with a fourth-order Runge-Kutta method. The effects of the governing dimensionless parameters on the velocity profiles, temperature distribution, skin friction coefficient, and local Nusselt number are analyzed systematically. The physical insights gained from this study are expected to assist in the design and optimization of thermal processing systems involving electrically conducting non-Newtonian fluids.

Nomenclature

u, w	Velocity components	μ	Dynamic viscosity
r, z	coordinates	k	Thermal conductivity
t	Time	$\mathrm{C_p}$	Specific Heat at constant pressure
T	Fluid Temperature	α	Thermal diffusivity
T_∞	Ambient temperature	$\mathrm{B_0}$	Applied magnetic field strength
$\mathrm{T_w}$	Surface Temperature	σ	Electrical conductivity
P	Pressure	η	Similarity variable
ρ	Fluid Density	θ	Dimensionless temperature
q_r	Radiative heat flux	ψ	Stream function
h_f	Heat transfer coefficient	Ec	Eckert number
Re	Reynolds number	M	Magnetic parameter
Pr	Prandtl number	A	Unsteadiness parameter
Bi	Biot number	n	Power law index
S	Mass transfer parameter		

2. Mathematical Formulation

A power-law fluid is considered to flow over a stretching sheet with an unsteady, incompressible, axisymmetric magneto-hydrodynamic (MHD) motion. It is assumed that the flow is laminar and that the induced magnetic field is negligible (low magnetic Reynolds number). A cylindrical coordinate system is used here with the radial coordinate r parallel to the sheet and the axial coordinate z normal to the sheet. The schematic diagram and the coordinate system considered are shown inFig.1

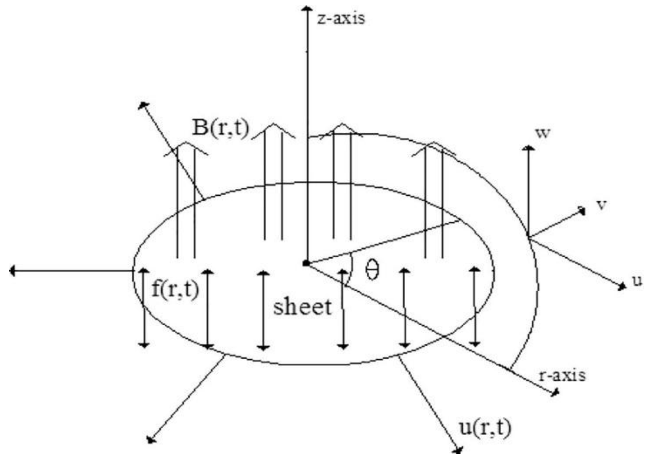

Fig.1. Flow configuration and coordinate system.

The conservation of mass, momentum, and energy equations under the boundary layer assumptions are given below:

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}=-\frac{K}{\rho}\frac{\partial}{\partial z}\left(-\frac{\partial u}{\partial z}\right)^{n}-\frac{\sigma B_0^2}{\rho}u \tag{2}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}=\alpha\frac{\partial^2 T}{\partial z^2}+\frac{K}{\rho C_p}\left|\frac{\partial u}{\partial z}\right|^{n+1}+\frac{\sigma}{\rho C_p}B_0^2u^2-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial z} \tag{3}$$

The radiative heat flux $\mathrm{q_r}$ is modeled using the Roseland approximation. By linearizing the temperature-dependent term, the energy equation reduces to:

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}=\left(\alpha+\frac{16\sigma^*}{3k^*}T_\infty^3\right)\frac{\partial^2 T}{\partial z^2}+\frac{1}{\rho C_p}K\left|\frac{\partial u}{\partial z}\right|^{n+1}+\frac{\sigma}{\rho C_p}B_0^2u^2 \tag{4}$$

The associated boundary conditions are given as:

$$\left.\begin{array}{c}\mathrm{u}=\mathrm{u_w(r,t)},\mathrm{w}=-\mathrm{f_w(r,t)}\,,\mathrm{k}\dfrac{\partial \mathrm{T}}{\partial \mathrm{z}}=-\mathrm{h_f}[\mathrm{T_f}-\mathrm{T}]\ \mathrm{at}\ \mathrm{z}=0,\\ \mathrm{u}\to 0\,,\mathrm{T}\to \mathrm{T_\infty}\ \ \mathrm{as}\ \mathrm{z}\to\infty.\end{array}\right\} \tag{5}$$

Suitable similarity transformations are introduced and the set of coupled partial differential equations governing the problem is transformed into a set of nonlinear ordinary differential equations for the reduced problem.

$$\psi(r,z) = -\frac{cr^3}{1-\alpha t} Re^{\frac{-1}{n+1}} f(\eta), \theta(\eta) = \frac{T - T_\infty}{T_W - T_\infty}, \eta = \frac{z}{r} Re^{\frac{1}{n+1}}, Re = \frac{r^n u_w^{2-n}}{k/\rho} \tag{6}$$

Using the stream function formulation and similarity variables, $u = -\frac{1}{r}\frac{\partial \psi}{\partial z}$, $w = \frac{1}{r}\frac{\partial \psi}{\partial r}$ the continuity equation is identically satisfied, while the momentum and energy equations are transformed into:

$$\left(| f''(\eta) |^{n-1} f''(\eta)\right)' + \frac{3n+1}{n+1} f(\eta) f''(\eta) - \left(f'(\eta)\right)^2 - A\left(f'(\eta) + \frac{\eta}{2} f''(\eta)\right) - M f'(\eta) = 0 \tag{7}$$

$$\left(\frac{1}{Pr} + \frac{4}{3R}\right)\theta''(\eta) + \frac{2n}{n+1} f(\eta)\theta'(\eta) - A\left(\frac{\eta}{2}\right)\theta'(\eta) + Ec\left(| f''(\eta) |^{n+1} + M\left(f'(\eta)\right)^2\right) = 0 \tag{8}$$

Along with transformed boundary conditions:

$$\left.\begin{matrix} f(0) = S, f'(0) = 1 \ and \ \theta'(0) = -Bi[1 - \theta(0)], & at \ \eta = 0 \\ f'(\infty) \to 0, \theta(\infty) \to 0, & as \ \eta \to \infty \end{matrix}\right\}. \tag{9}$$

Where $A = \alpha c$, $M = \frac{\sigma B_0^2(1-\alpha t)}{\rho c}$, $Pr = \frac{\rho C_p}{k}\left(\frac{K}{\rho}\right)^{\frac{2}{n+1}}\left(r^{\frac{2(n-1)}{n+1}}\right)\left(U_W^{\frac{3(n-1)}{n+1}}\right)$, $R = \frac{3k^* k}{16\sigma^*} T_\infty^3$, $Ec = \frac{u_W^2}{C_p(T_W - T_\infty)}$, $Bi = \frac{h_f r}{k} Re^{-\frac{1}{n+1}}$

The important physical quantities of interest in this problem are the local skin friction coefficient C_f and the local Nusselt number Nu at the walls defined as:

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho u_w^2}, \qquad Nu = \frac{q_w r}{k}\left(T_f - T_w\right),$$

where $\tau_w = K\left(-\frac{\partial u}{\partial z}\right)^n\Big|_{z=0}$ and $q_w = -\left(k + \frac{16\sigma^* T_\infty^3}{3k^*}\right)\frac{\partial T}{\partial z}\Big|_{z=0}$ are the shear stress at the wall and the term $\frac{16\sigma^* T_\infty^3}{3k^*}$is the radiative conductivity.

$$\frac{1}{2} C_f \, Re^{\frac{1}{n+1}} = (-1)^{n+1}\left(f''(0)\right)^n.$$

The quantity $C_f \, Re^{\frac{1}{n+1}} =$ is the dimensionless skin friction coefficient or the local friction factor.

$$NuRe^{-\frac{1}{n+1}} = -\left(1 + \frac{1}{R}\right)\theta'(0)$$

The quantity $NuRe^{-\frac{1}{n+1}}$ is the dimensionless Nusselt number or the reduced Nusselt number.

3. Numerical Method

The obtained system of nonlinear ODEs with the corresponding boundary conditions are numerically solved by the shooting method and fourth order Runge-Kutta scheme. The shooting method involves making a transformation of the boundary value problem into an initial value problem, using appropriate initial conditions, which are then iteratively updated until the boundary conditions at infinity are met. The similarity transformations are then performed and the partial differential equations governing the problem are subsequently transformed into a coupled set of nonlinear ordinary differential equations (ODEs) for the stream function formulation. The governing partial differential equations (PDEs) are then simplified to a coupled system of nonlinear ordinary differential equations (ODEs) for the stream function formulation after performing the similarity transformations. Let's consider

$$y_1 = f, y_2 = f', y_3 = f'', yy_1 = f'''$$

So, momentum equation (7) becomes:

$$yy_1 = \frac{1}{n|y_3|^{n-1}}\left[A\left(y_2 + \frac{\eta}{2}y_3\right) + My_2 + y_2^2 - \left(\frac{3n+1}{n+1}\right)y_1y_3\right]$$

Also,

$$y_4 = \theta, y_5 = \theta', yy_2 = \theta''$$

So equation (8) takes the form:

$$\left(\frac{1}{Pr} + \frac{4}{3R}\right)yy_2 + \frac{2n}{n+1}y_1y_5 - A\left(\frac{\eta}{2}\right)y_5 + Ec(|\ y_3\ |^{n+1} + M(y_2)^2) = 0$$

The corresponding boundary conditions can be written as:

$$\left.\begin{array}{c} y_1(0) = S\,, y_2(0) = 1 \; and \; y_5(0) = -Bi[1 - y_4(0)], \qquad at\; \eta = 0 \\ y_2(\infty) \to 0\,, y_4(\infty) \to 0, \qquad as\; \eta \to \infty \end{array}\right\}.$$

To solve nonlinear systems, the Runge–Kutta method is used for numerical integration because it is accurate and stable. The two methods in combination offer an efficient and reliable solution to the problem at hand, especially when the flow domain is a semi-infinite region. In recent researches [22] similar numerical methods have been successfully used to solve nonlinear MHD non-Newtonian fluid flow problems.

The numerical values of the skin friction coefficient and local Nusselt number are calculated and presented in Table 1 and 2 which are quantitative values of the flow and heat transfer characteristics. These parameters have a considerable influence on the description of shear stress at the surface and the transfer rate of heat, respectively.

Table 1: Numerical values of skin friction coefficient for different values of the physical parameters.

Physical parameters				**Skin Friction** $-\frac{1}{2}Re^{\frac{1}{n+1}}C_f$		
M	S	Nr	Bi	$n = 0.8$	$n = 1.0$	$n = 1.2$
0.0	−	−	−	0.925013	1.446721	1.052148
0.5	−	−	−	0.899382	1.662144	1.075391
1.0	−	−	−	0.879552	1.675595	1.094454
−	0.0	−	−	0.893434	1.541421	1.081903
−	0.5	−	−	0.879552	1.739052	1.094454
−	1.0	−	−	0.867289	1.965710	1.106115
−	−	0.0	−	0.879552	1.646829	1.094454
−	−	0.5	−	0.879552	1.646829	1.094454
−	−	1.0	−	0.879552	1.646829	1.094454
−	−	−	0.0	0.879552	1.646829	1.094454
−	−	−	0.5	0.879552	1.646829	1.094454
−	−	−	1.0	0.879552	1.646829	1.094454

Table 2: Numerical values of the local Nusselt number for various values of the physical parameters.

Physical parameters				Nusselt Number		
M	S	Nr	Bi	$n = 0.8$	$n = 1.0$	$n = 1.2$
0.0	−	−	−	0.327446	0.668433	0.317908
0.5	−	−	−	0.307969	0.667323	0.298615
1.0	−	−	−	0.288747	0.673064	0.279647
−	0.0	−	−	0.334201	0.657301	0.326954
−	0.5	−	−	0.288747	0.645632	0.279647
−	1.0	−	−	0.217258	0.625252	0.2125511
−	−	0.0	−	0.314488	0.314382	0.304970
−	−	0.5	−	0.288747	0.2899614	0.279647
−	−	1.0	−	0.265674	0.267129	0.256105
−	−	−	0.0	0.000000	0.000000	0.000000
−	−	−	0.5	0.288747	0.289614	0.279647
−	−	−	1.0	0.441159	0.445198	0.429284

With the increase of magnetic parameter, it is noticed that the skin friction coefficient increases because the resistive force increases and the rate of heat transfer decreases with the decrease of Nusselt number. This is also observed for other governing parameters, which show that they have a strong influence on the flow and thermal properties.

Table 3. Validation of the Present Numerical Results with Ahmed et al. (2017) for n=1.0

Parameters	Values	Ahmed *et al.* [10] Skin Friction	Present study Skin Friction	Ahmed *et al.* [10] Nusselt Number	Present study Nusselt Number
M	0.0	1.45257	1.44672	0.66851	0.66843
	0.5	1.63582	1.66214	0.66651	0.66732
	1.0	1.79894	1.67595	0.66476	0.67306
S	0.0	1.57235	1.54142	0.66157	0.65730
	0.5	1.75664	1.73905	0.63633	0.64563
	1.0	1.92178	1.96571	0.60995	0.62525

For a comparison of the present numerical results with those of Ahmed et al. (2017) for the common governing parameters, this comparison is shown in Table 3. The present mathematical model considers the additional physical effects such as thermal radiation, viscous dissipation, joule heating and convective boundary condition, but the numerical trends of both skin friction coefficient and local Nusselt number is seen to be in accordance with the published results. The quantitative differences observed are due to the extra thermal mechanisms accounted for in the present formulation, thus validating the reliability and accuracy of the present numerical solution.

Graphical results and discussion

The present work deals with the unsteady axisymmetric MHD flow of a power-law fluid past a stretching sheet with consideration of viscous dissipation, joule heating and thermal radiation. Major parameters affecting the flow and heat transfer are (M) magnetic parameter, (Ec) Eckert number, (R) radiation parameter, (Pr) Prandtl number and (A) unsteadiness parameter. Magnetic field exerts on fluid a Lorentz force which opposes fluid motion and influences velocity and temperature. The viscous dissipation (Ec) can raise the temperature by transferring the kinetic energy as heat, whereas the thermal radiation (R) can improve the energy transport. The relative thickness of the thermal and momentum boundary layers is determined by a Prandtl number (Pr), unsteadiness (S) represents the time-dependent effects, and the power-law index (n) describes the rheology of the fluid. The following graphical results illustrate the changes in the above mentioned parameters in relation to velocity, temperature, wall shear stress and heat transfer with numerical solutions.

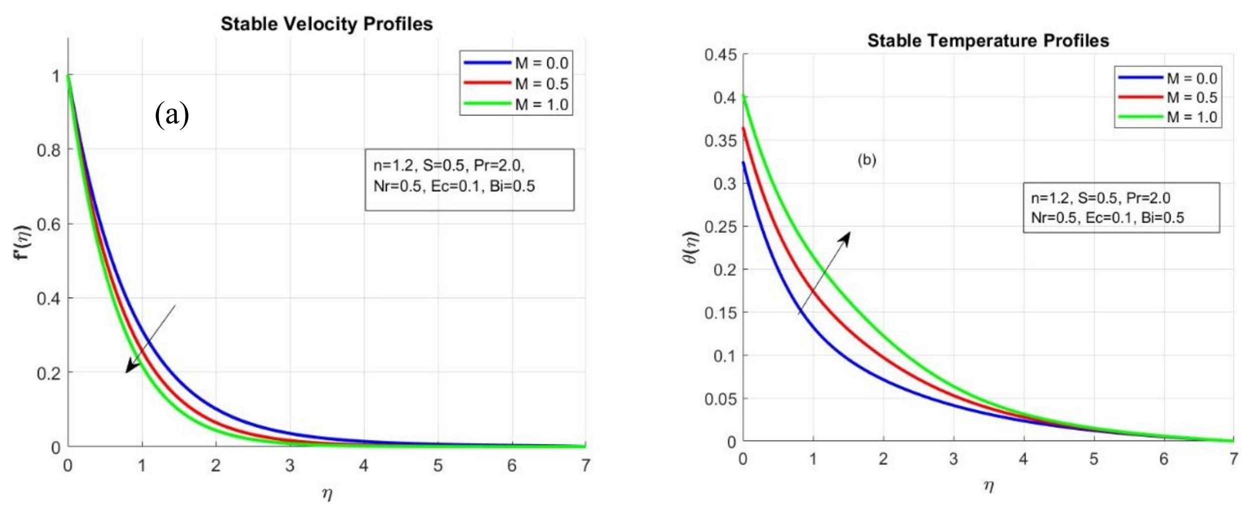

Figure 2(a-b): Effect of magnetic parameter (M) on velocity and temperature profile.

Figure 2(a) shows that increasing the magnetic parameter reduces the fluid velocity due to the Lorentz force opposing the motion. The resistive effect becomes stronger with higher M, slowing down the flow.

Figure 2(b) indicates that higher magnetic parameters increase the temperature within the boundary layer. The Joule heating generated by the magnetic field elevates thermal energy, thickening the thermal boundary layer. Similar thermal enhancement due to magnetic field effects has been reported in related MHD heat-transfer studies [24].

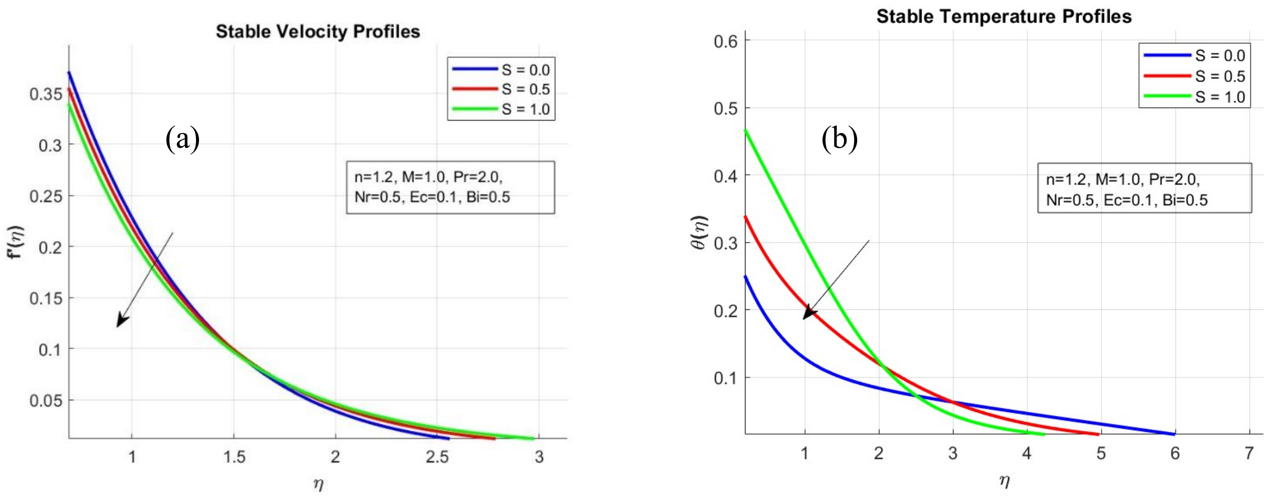

Figure 3(a-b): Effect of unsteadiness parameter (S) on velocity and temperature profile.

Figure 3(a) shows that increasing the unsteadiness parameter reduces the velocity near the sheet. A higher S intensifies the time-dependent deceleration, suppressing the momentum within the boundary layer.

Figure 3(b) indicates that higher values of the unsteadiness parameter slightly increase the temperature profile. The slower fluid motion due to unsteadiness enhances thermal energy retention, thickening the thermal boundary layer.

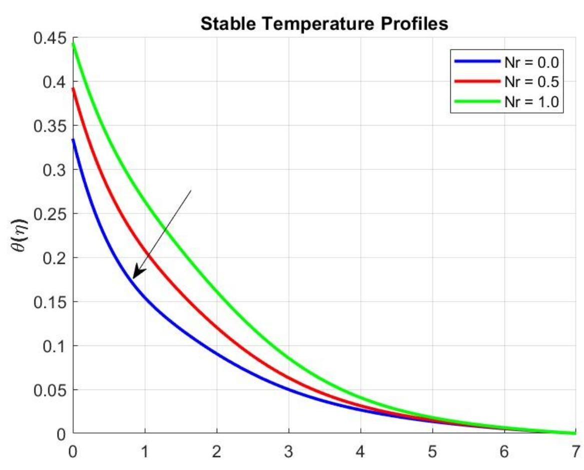

Figure 4: Effect of Thermal radiation parameter (Nr) on temperature profile.

Figure 4 shows the impact of the thermal radiation parameter on the temperature profiles. It is evident that increasing the radiation parameter enhances the temperature distribution within the boundary layer due to the additional radiative heat flux. The increase in temperature with thermal radiation agrees well with previously published results on radiative MHD flows of power-law fluids [25].

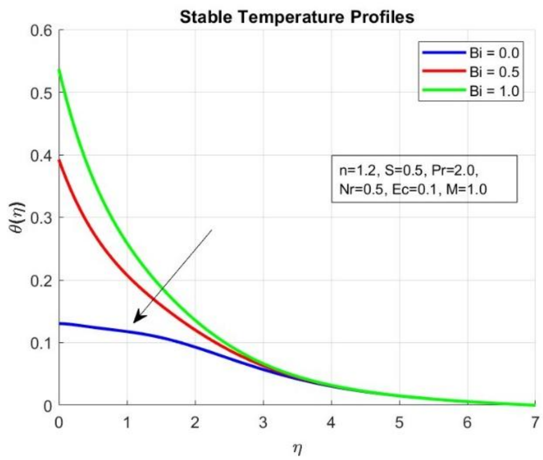

Figure 5: Effect of Biot Number parameter (Bi) on temperature profile.

The figure 5 shows that as the Biot number increases, the heat transfer from the stretching sheet to the fluid is increased. A higher value of higher Bi represents more convective effects at the surface which leads to a steeper temperature gradient and thinner thermal boundary layer. Such behavior is similar to the observations that have been reported for convective heat transfer in non-Newtonian fluids with different Biot numbers [23].

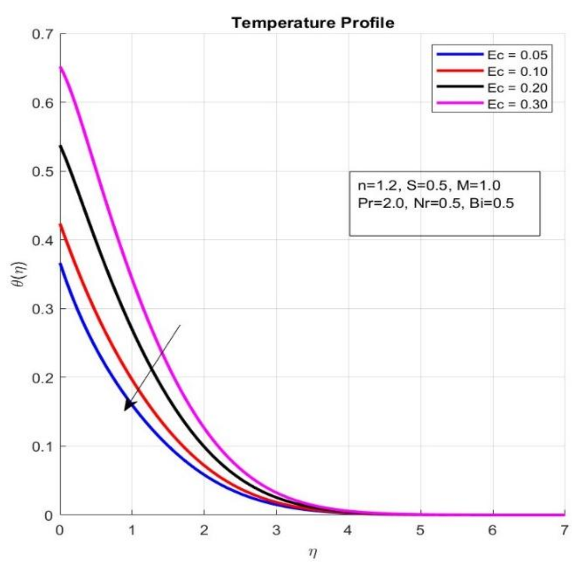

Figure 6: Effect of Eckert Number parameter (Ec) on temperature profile.

Figure 6 shows the effect of the Eckert number (Ec) on the flow and thermal fields. The temperature profile rises with higher Ec due to enhanced viscous dissipation, which increases heat generation within the fluid.

Numerical solutions to the transformed ODEs of the MHD axisymmetric flow of a power-law fluid over an unsteady stretching sheet are shown graphically and tabulated values of the skin friction coefficient and Nusselt number for the flow are obtained. The influence of the various governing parameters like magnetic parameter (M), Eckert number (Ec), radiation parameter (R), Prandtl number (Pr), power-law index (n), unsteadiness parameter (S) and Biot number (Bi) are analyzed to understand the effect on the flow and thermal characteristics.

Concluding Remarks

The unsteady, axisymmetric MHD flow of a power-law fluid over a stretching sheet with convective boundary conditions has been numerically studied. The effects of magnetic parameter, Eckert number, radiation parameter, Prandtl number, power-law index, unsteadiness and Biot number on velocity, temperature, skin friction and heat transfer have been studied systematically.

The results indicate a significant suppression of the fluid motion and an enhancement of the thermal energy by the Joule heating due to the magnetic field. The viscous dissipation and thermal radiation increase the temperature in the boundary layer and the higher Prandtl numbers and unsteadiness parameters influence the thermal boundary layer thickness. The power-law index, which classifies the shear-thinning and shear-thickening fluids, greatly affects the flow behavior. The Biot number governs the convective effects on the surface heat transfer.

The results give a useful insight into the complicated interactions of flow and thermal fields in non-Newtonian fluids under the influence of MHD and thermal radiation. The present analysis offers a robust and flexible framework that can be generalized to optimize industrial processes, thermal management systems and materials processing applications involving power-law fluids.

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