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Joule and Viscous Dissipation Effects on MHD Boundary Layer Flow over a Stretching Sheet with Variable Thickness

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Abstract

This paper is aimed to investigate the influence of Joule and viscous dissipation effects on boundary layer flow over a stretching sheet with variable thickness and surface temperature. The said flow is subjected to space dependent magnetic field applied normal to the sheet. Mathematical modeling is done under boundary layer approximations. The governing partial differential equations are transformed into ordinary differential equations via appropriate similarity transformations. The resulting set of nonlinear equations is solved numerically using shooting method. MATLAB software is used to obtain numerical results. The impact of various physical parameters, such as power index, magnetic parameter, on velocity and temperature profiles is analyzed. Also, their effects on skin friction coefficient and Nusselt number are presented and discussed.

Keywords: *Boundary Layer Flow; Heat Transfer Rate; MHD; Skin Friction; Stretching Sheet; Variable Thickness.*

1. Introduction

The study of fluid flow over a stretching surface is a vital problem discussed in current era as it occurs in different engineering processes such as melt-whirling, wire drawing, production of glass fiber, extrusion, manufacturing of rubber sheets and cooling of huge metallic plates such as an electrolyte [1]-[3]. The first study of boundary layer flow on continuous solid surface was carried out by [4]. Further extension of his work was done by [5] in which the surface velocity is related to the distance from flat surface.
Magnetohydrodynamic boundary layer flow is of considerable interest as it is widely used in geothermal applications and industrial technology, high temperature plasmas, MHD power generation systems and liquid metal fluids. Many metallurgical processes depend significantly on the rate of cooling of product. To control cooling rate, the sheets are stretched and passed through electrically conducting fluid in the presence of magnetic field. In view of all these aspects, many researchers have investigated the magnetic field effect on the fluid flow problems. Chakrabarti and Gupta [6], and Jhankal and Kumar [7] worked on heat transfer characteristics over a stretching sheet with hydromagnetic flow. Sparrow and Cess [8] analyzed free convection phenomena with magnetic effects. Afterwards, much emphasis has been given on transversely applied magnetic field on electrically conducting fluids. Chaim [9] carried out his work on transverse magnetic field applied on stretching sheet with suction and blowing effects in the presence of electrically conducting fluid.
In real world applications, the stretching sheets are not necessarily flat, rather they can have variable thickness. The stretching sheets with variable thickness are used more frequently in architecture, machine design, nuclear reactor technology, naval structures and acoustical components. Recently, the boundary layer flow on a stretching sheet with variable thickness has been investigated by Fang et al. [10]. Also, Khader and

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Megahed [11] obtained the numerical solutions for boundary layer flow of a nonlinearly stretching sheet having variable thickness. Moreover, the concept about variable surface temperature becomes vital in the study of deforming objects like stretching sheet having variable thickness slandering away from slot. The heat transfer characteristics of stretching surface with variable temperature were investigated by Grubka and Bobba [12]. Devi and Thiyagarajan [13] worked on hydromagneic flow past a stretching sheet with variable surface temperature. The same work was examined by Devi and Prakash [14] with stretching sheet of variable thickness.
The effects of Joule and viscous dissipation on the MHD flow and heat transfer are worth studying because of their abundant applications. It plays a significant role in various heating devices which are being prepared by metallurgical processes and extrusion. Also, crystal growing, cooling down of electronic chips and metallic sheets, paper production, glass fibre drawing etc. depend on rate of heating or cooling. Viscous dissipation behaves like some source of energy as it alters the temperature profiles, consequently, changes the rate of heat transfer. Therefore, many researchers like Jaber [15], Palani and Kim [16], Hamid and Arifin [17], Devi and Ganga [18], studied the effects of viscous dissipation and Joule heating on stretching plates or surfaces with variable properties.
In most of the recent works on stretching sheets, for instance [19], [20], [21], not much emphases was given to investigate the effects of Joule and viscous dissipation on MHD boundary layer flow over a stretching sheet along with variable thickness and variable surface temperature. Hence, this work focuses to investigate such flows and it is expected that the present work will add significant advancement to the existing knowledge.

2. Formulation of the Problem

Consider MHD boundary layer flow of an electrically conducting fluid past a stretching sheet with variable thickness and variable surface temperature. It is assumed that flow is steady, laminar, and incompressible. Joule heating and viscous dissipation effects are significantly taken into consideration. A variable magnetic field is applied normal to the surface. The x-axis is chosen parallel to stretching sheet and y-axis lies perpendicular to it.

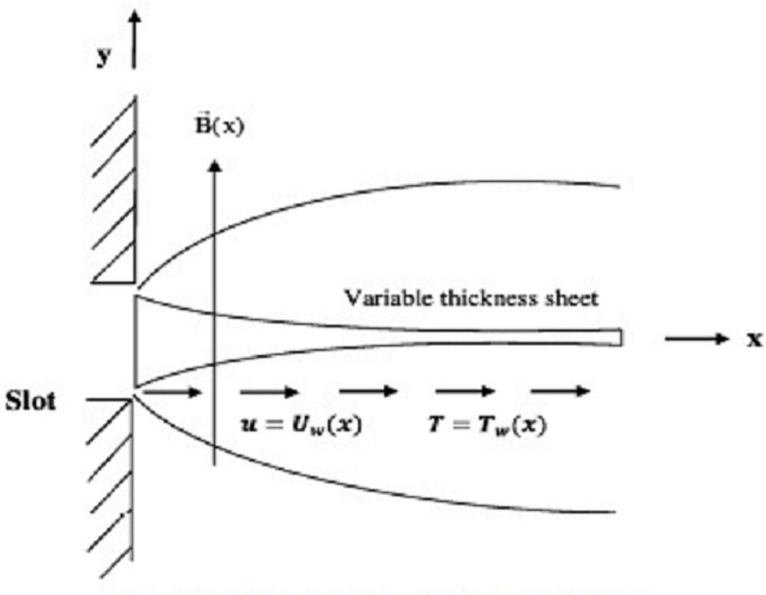


Figure 2.1: Schematic diagram of a stretching sheet with variable thickness

The following points are assumed to study the above mentioned problem.

(i) The wall is assumed to be non-porous with $v_w = 0$.

(ii) The sheet of variable thickness follows the profile given by

$$y = A(x+b)^{\frac{1-m}{2}}, (m \neq 1) \tag{2.1}$$

where m is the velocity power index and b represents stretching sheet parameter.

(iii) The sheet is being stretched with power law velocity

$$U_w(x) = U_0(x+b)^m, \tag{2.2}$$

where U_0 is constant.

(iv) In order to neglect induced magnetic field, the small value of magnetic Reynolds number is considered. Also, the external electric field is neglected as $\nabla \cdot \vec{E} = 0$ as well as $\nabla \times \vec{E} = 0$.

(v) The surface wall temperature is considered as

$$T_w(x) = T_\infty + T_0(x+b)^r, \tag{2.3}$$

where r gives the temperature index parameter.

(vi) The special form of transverse magnetic field is given by

$$B(x) = B_0(x+b)^{\frac{m-1}{2}}. \tag{2.4}$$

Under the above assumptions and boundary layer approximations for Newtonian fluid, the basic equations of continuity, Navier-Stokes, and energy take the following form ([14] with viscous dissipation and Joule heating effects):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2.5}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \sigma\frac{B(x)^2}{\rho}u, \tag{2.6}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\left(\frac{\partial^2 T}{\partial y^2}\right) + \frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{1}{\rho C_p}\left(\frac{1}{\sigma}\vec{J}\cdot\vec{J}\right), \tag{2.7}$$

where

$\vec{J} = \sigma(\vec{V} \times \vec{B})$.

The last two terms in eq.(2.7) comes into view due to viscous dissipation and Joule heating effects respectively. The associated boundary conditions are given by

$$u = U_w(x), \quad v = 0, \quad T = T_w(x), \quad \text{at} \quad y = A(x+b)^{\frac{1-m}{2}}, \tag{2.8}$$

$$u \longrightarrow 0, \quad \text{as} \quad y \longrightarrow \infty, \tag{2.9}$$

$$T \longrightarrow T_\infty, \quad \text{as} \quad y \longrightarrow \infty. \tag{2.10}$$

where $T_w(x)$ refers to temperature of wall, and T_∞ the surrounding temperature.

3. Analysis

Following Devi and Prakash [14], the following similarity function and similarity transformations are used.

$$\psi(x,y) = f(\eta)\sqrt{\frac{2}{m+1}\nu U_0(x+b)^{m+1}}, \tag{3.1}$$

$$\eta = y\sqrt{\frac{m+1}{2}\frac{U_o(x+b)^{m-1}}{\nu}}, \quad (m \neq 1), \tag{3.2}$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w(x) - T_\infty}. \tag{3.3}$$

The equation of continuity is justified for the velocity components given by

$$u = \frac{\partial \psi}{\partial y}, \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x}. \tag{3.4}$$

By utilizing similarity transformations (3.1-3.3), the nonlinear partial differential equations (2.6-2.7) are transformed to the following nonlinear ordinary differential equations:

$$f''' = \left[\left(\frac{2m}{m+1}\right)(f')^2 - ff'' + M^2 f'\right], \tag{3.5}$$

$$\theta'' = Pr\left[\left(\frac{4m}{m+1}\right)f'\theta - f\theta'\right] - Pr\cdot Ec\left[(f'')^2 + M^2(f')^2\right]. \tag{3.6}$$

with the boundary conditions,

$$f(\alpha)=\alpha\left(\frac{1-m}{m+1}\right), \qquad f'(\alpha)=1, \qquad \theta(\alpha)=1, \tag{3.7}$$

$$f'(\infty)=0, \qquad \theta(\infty)=0. \tag{3.8}$$

Here $\alpha=A\sqrt{\frac{m+1}{2}\frac{U_0}{\nu}}$ represents wall thickness parameter and $\eta=\alpha$ indicates the plate surface. To make computations easy and domain $[0,\infty)$, certain functions are introduced i.e. $F(\xi)=F(\eta-\alpha)=f(\eta)$ and $\theta(\xi)=\theta(\eta-\alpha)=\theta(\eta)$. Hence, the equations (3.5) and (3.6) take the following form:

$$F'''=\left[\left(\frac{2m}{m+1}\right)(F')^2-FF''+M^2F'\right], \tag{3.9}$$

$$\theta''=Pr\left[\left(\frac{2r}{m+1}\right)F'\theta-F\theta'\right]-Pr\cdot Ec\left[(F'')^2+M^2(F')^2\right]. \tag{3.10}$$

The related boundary conditions become:

$$F(0)=\alpha\left(\frac{1-m}{m+1}\right), \qquad F'(0)=1, \qquad \theta(0)=1, \tag{3.11}$$

$$F'(\infty)=0, \qquad \theta(\infty)=0. \tag{3.12}$$

Here, differentiation is carried out with respect to ξ. Also, $Pr=\frac{\mu C_p}{k}$ refers to Prandtl number, and $M^2=\frac{2\sigma B_0^2}{\rho U_0(m+1)}$ corresponds to the magnetic interaction parameter. Also, viscous dissipation is determined by Eckert number while the product of Eckert number and magnetic paramter gives Joule heating.
The important physical quantities, namely; local skin-friction coefficient C_f and the local Nusselt number Nu_x, are given by:

$$C_f=\frac{\mu\left(\frac{\partial u}{\partial y}\right)_{y=A(x+b)^{\frac{1-m}{2}}}}{\frac{1}{2}\rho U_w^2}=2\sqrt{\frac{m+1}{2}}(Re_x)^{\frac{-1}{2}}F''(0), \tag{3.13}$$

$$Nu_x=\frac{(x+b)\left(\frac{\partial T}{\partial y}\right)_{y=A(x+b)^{\frac{1-m}{2}}}}{T_w(x)-T_\infty}=-\sqrt{\frac{m+1}{2}}(Re_x)^{\frac{1}{2}}\theta'(0). \tag{3.14}$$

Here $Re_x=\frac{U_wX}{\nu}$ shows the Reynolds number and $X=(x+b)$.

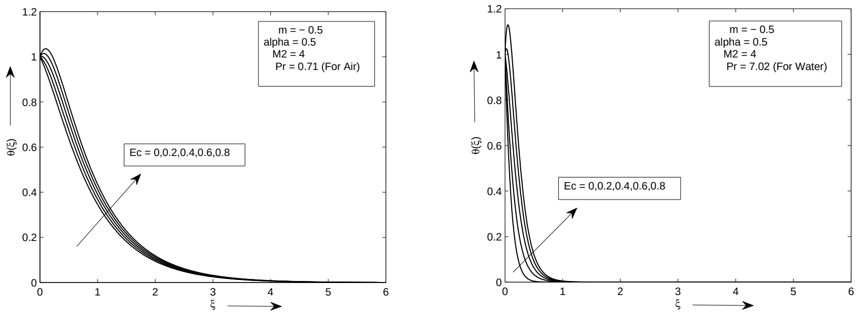
4. Numerical Solution

The set of nonlinear ordinary differential equations (3.9-3.10) together with boundary conditions (3.11-3.12) are solved by using shooting method [22]. The shooting method is based on converting a boundary value problem into an initial value problem. In order to get accurate results of initial value problem, a suitable initial guess is required to start a recursive procedure. For present problem, initial guesses are required for the values of $F''(0)$ and $\theta'(0)$. MATLAB built-in solver 'ode45' is used to solve initial value problem which involves variable step size Runge-Kutta method. Afterwards, MATLAB function 'fsolve' is executed to improve the initial guess to approach terminal conditions. The process is repeated till the required results are obtained up to the accuracy of 10^{-6} level, which is desirable to meet convergence criteria.

5. Results and Discussion

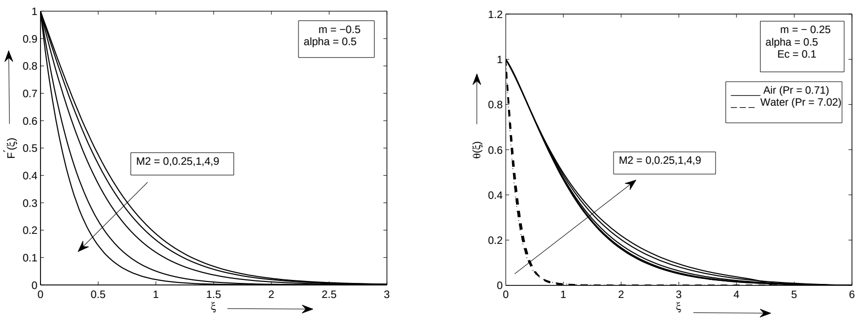
In the present work, our main interest is to study the consequences of Joule and viscous dissipation effects on velocity and temperature profiles of given problem. Under these special effects, the influence of different physical parameters like magnetic interaction, wall thickness, velocity power and temperature indices, over a stretching sheet with variable thickness has been examined and discussed. Furthermore, the

graphs of dimensionless coefficient of skin friction as well as non-dimensional heat transfer rate against various parameters are plotted and analyzed.
As Eckert number (Ec) plays significant role in Joule and viscous dissipation effects, so firstly its effect is observed on temperature distribution through Figure 5.1. It is examined that increasing the value of Eckert number (Ec) enhances the temperature profile. The same behaviour is observed for both air $(Pr = 0.71)$ and water $(Pr = 7.02)$. Also, it is evident that the thermal boundary layer gets thinner for higher value of Prandtl number.



(a) When $Pr = 0.71$ (for air). **(b)** When $Pr = 7.02$ (for water).

Figure 5.1: Variation of temperature profile against various values of Ec.



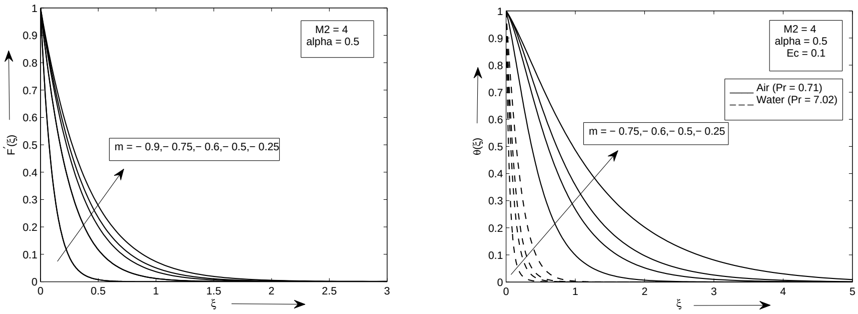
(a) Velocity Profile **(b)** Temperature Profile

Figure 5.2: Behavior against various values of magnetic parameter.

Figure 5.2 refers to the effect of magnetic parameter (M^2) on velocity profile and temperature distribution in the boundary layer. It is observed that momentum boundary layer thickness reduces with increase in the value of M^2 whereas temperature shows an increasing behaviour. As magnetic parameter comes into sight because of Lorentz force which is resistive force, so it tends to impede the motion of fluid which supports our results. Also, the resistance on fluid particles increases which results in the increase of temperature. Moreover, it is observed that magnetic parameter affects more significantly in case of air $(Pr = 0.71)$ than water $(Pr = 7.02)$.

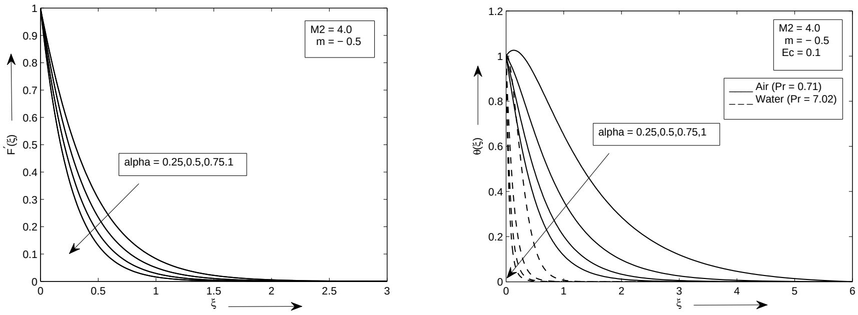
Figure 5.3 shows the effect of velocity power index parameter (m) on velocity and temperature distributions. Increase in the values of velocity power index causes velocity as well as temperature profile to increase. Furthermore, thermal boundary layer can be reduced by using higher values of Prandtl number.
The influence of wall thickness parameter (α) on velocity and temperature profiles can be seen through Figure 5.4. It is inferred that stretching of sheet away from a slit leads to decrease the wall thickness. By decreasing wall thickness, the velocity and temperature profiles gets reduced. The thermal boundary layer gets thinner for large values of wall thickness parameter. Furthermore, it is noticed that thermal boundary layer for air is thicker as compared to water.

The variation of skin friction coefficient against different parameters involved is investigated through Figure 5.5. By raising the values of wall thickness and magnetic parameters, the skin friction coefficient increases in magnitude. Moreover, its variations is examined for different values of velocity power index and wall thickness parameter. It comes into sight that for $\alpha \leq 0.25$, the friction coefficient gets decreased but for $\alpha > 0.25$, it gets increased for growing values of velocity power index.



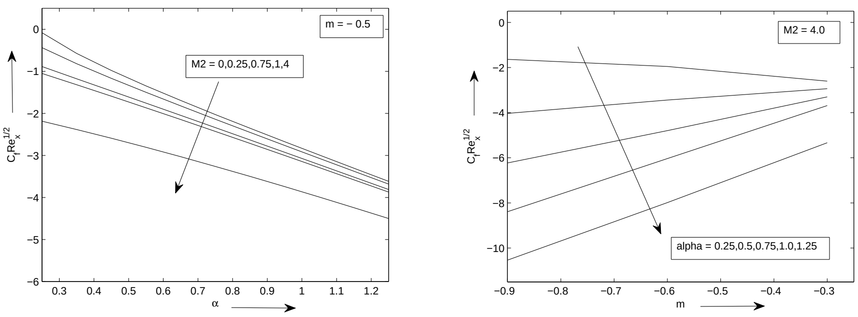
(a) Velocity Profile **(b)** Temperature Profile

Figure 5.3: Behavior against various values of velocity power index



(a) Velocity Profile **(b)** Temperature Profile

Figure 5.4: Behavior against various values of wall thickness parameter



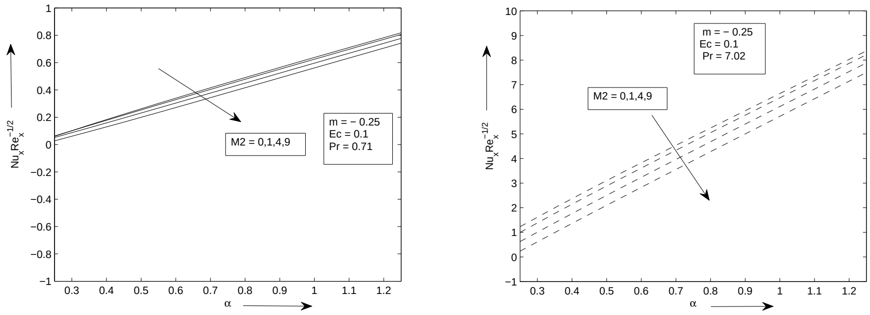
(a) For various values of M^2 **(b)** For various values of α

Figure 5.5: Variation of dimensionless skin friction coefficient

Figure 5.6 illustrates the influence of various parameters on non-dimensional heat transfer rate for both air $(Pr = 0.71)$ and water $(Pr = 7.02)$. It is noticed that heat transfer rate gets reduced as magnetic parameter (M^2) is increased. The reverse trend is followed against wall thickness parameter (α) i.e. rate of heat transfer increases for higher values of wall thickness parameter. Moreover, more heat transfer rate is observed for water than air.
The effect of velocity power index (m) on non-dimensional heat transfer rate can be seen through figure 5.7. It is observed that increase in the value of velocity power index causes decrease in heat transfer rate. Also, more heat transfer rate is observed for water $(Pr = 7.02)$ as compared to air $(Pr = 0.71)$.

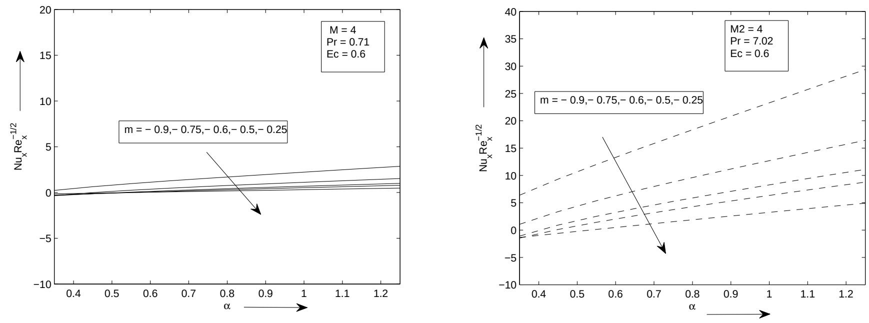
Comparison of heat transfer rate for different values of velocity power index (m) can be seen through table 1. It is observed that by introducing Joule and viscous dissipation effects, there is more reduction in heat transfer rate as compared to the rate without these effects.

The variation in heat transfer rate for different values of Eckert and Prandtl number is examined by figure 5.8. It can be clearly seen that as Eckert number gains higher values, it causes decrease in transfer rate.



(**a**) For $Pr = 0.71$ (for air) (**b**) For $Pr = 7.02$ (for water)

Figure 5.6: Variation of dimensionless heat transfer rate against α for various values of M^2



(**a**) For $Pr = 0.71$ (for air) (**b**) For $Pr = 7.02$ (for water)

Figure 5.7: Variation of dimensionless heat transfer rate against α for various values of m

Table 1: Comparison for different values of velocity power index with $\alpha = 1.25$, $M^2 = 4$.

m	Pr	$-\theta'(0)$ for $Ec = 0$	$-\theta'(0)$ for $Ec = 0.6$	Pr	$-\theta'(0)$ for $Ec = 0$	$-\theta'(0)$ for $Ec = 0.6$
-0.9		3.8134	2.4858		37.3714	25.543
-0.75		2.2777	1.2839		21.8920	4.9371
-0.6	0.71	1.7056	0.7924	7.02	15.9630	9.0050
-0.5		1.47305	0.5871		13.4953	6.7574
-0.25		1.1040	0.1760		9.4838	2.9205

Moreover, further reduction is noticed for large values of Prandtl number. Hence, the rate falls down more rapidly as Prandtl number gets higher values which can clearly be seen through table 2.

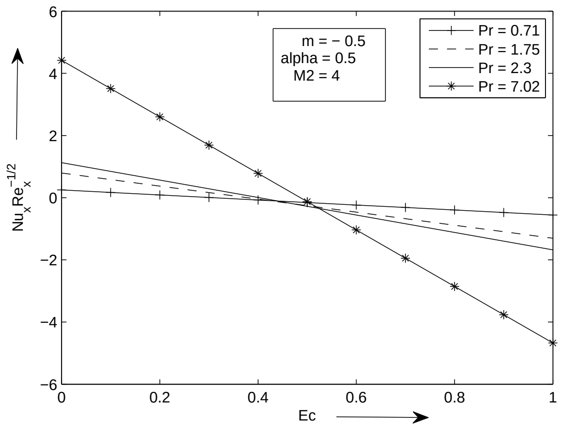


Figure 5.8: Variation in heat transfer rate against Ec for various values of Pr

Table 2: Effect of Eckert number on dimensionless heat transfer rate for $m = -0.5$, $\alpha = 0.5$ and $M^2 = 4$

Pr	Ec	$-\theta'(0)$	Pr	Ec	$-\theta'(0)$
0.71	0	0.24785	7.02	0	4.41685
	0.2	0.08655		0.2	2.59885
	0.6	-0.2360		0.6	-1.03715
	0.8	-0.3973		0.8	-2.85515

6. Conclusion

This research work covers the behaviour of MHD boundary flow on a stretching sheet having variable thickness and variable surface temperature with the emphasis on Joule and viscous dissipation effects. Mathematical model is formulated under boundary layer approximations and certain assumptions. In the absence of Joule and viscous dissipation effects, i.e $Ec = 0$, the results are in excellent agreement with that of Devi and Prakash [14]. A new insight in the field is generated through this work and it adds significant advancement to the knowledge in the field. The important conclusions of study are summarized below:

(**i**) The velocity profile shows an increment for increasing velocity power index, whereas, it gets reduced by increasing magnetic interaction and wall thickness parameter.

(**ii**) The increasing variation of magnetic parameter and velocity power index leads to increase temperature profile, but it gets decreased for large wall thickness and temperature index parameter. The same trend is followed for air $(Pr = 0.71)$ as well as for water $(Pr = 7.02)$.

(**iii**) Increase in magnitude of dimensionless skin friction coefficient is observed for increasing values of magnetic and wall thickness parameter. Also, it gets reduced with increasing velocity power index for $\alpha \leq 0.25$ and gets increased for $\alpha > 0.25$

(**iv**) As wall thickness parameter and temperature power index get higher values, they enhance the non-dimensional heat transfer rate, whereas, velocity index and magnetic interaction factors suppress the transfer rate.

(**v**) Joule and viscous dissipation effects significantly reduces the rate of heat transfer. The growing values of Eckert number with higher values of Prandtl number and magnetic parameter rapidly drops down the transfer rate.

Nomenclature

α	Wall thickness parameter
η	Similarity variable
μ	Coefficient of viscosity
ψ	The stream function
ρ	Density of the fluid
σ	Electrical Conductivity
θ	Dimensionless temperature
B	The magnetic field
b	Stretching sheet parameter
C_f	local skin-friction coefficient
C_p	Specific heat at constant pressure
Ec	Eckert number
k	Thermal Conductivity
m	Velocity power index
M^2	Magnetic interaction parameter
Nu	Nusselt number
Pr	Prandtl Number
Re	Reynolds number
T	Temperature of the fluid
T_∞	Temperature of the fluid at infinity
T_w	Surface wall temperature
u	Component of velocity in x-direction
v	Component of velocity in y-direction

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