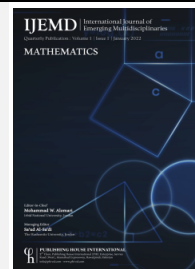




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## Autoregressive Distributed Lag Transformation for Exchange Rate and Trade Balance

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### Abstract

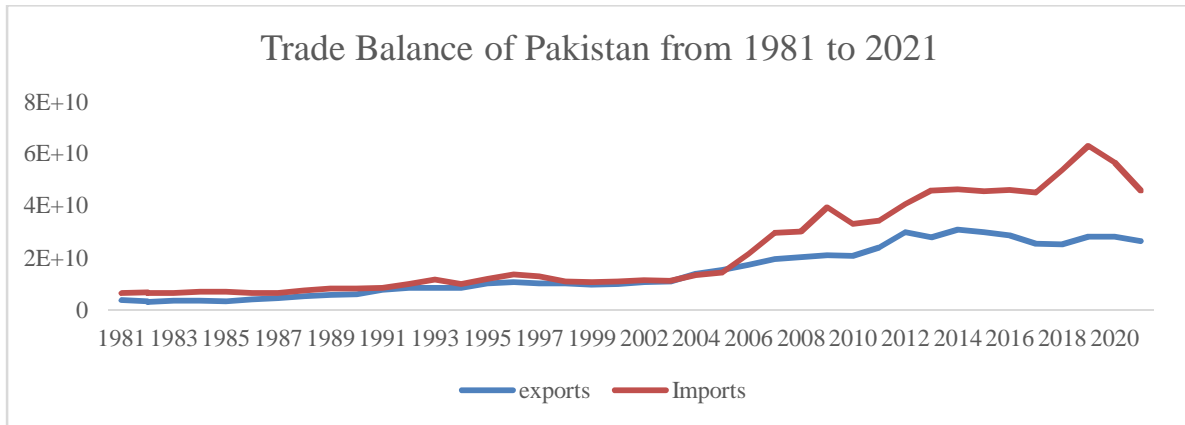
In this study the prime objective is to initiate autoregressive distributed lag transformation for exchange rate and trade balance to avoid the possible existence of multicollinearity among the explanatory variables and to analyze the variation in real exchange rate and its impact on trade balance in Pakistan. The Koyck model was used for studying the immediate impact and long run relationship between the variables by using time series annual data ranging from 1981 to 2021. The result showed that there is an inverse association between the trade balance and the exchange rate. It is evident from the results that the depreciation in exchange rate will bring upward movement in the trade balance.

**Keywords:** Autoregressive; Distributed lag; Exchange Rate; Trade Balance; Koyck Transformation; Pakistan

**2010 Mathematics Subject Classification:** 91B84; 62P20; 62M10; 62P05

### 1. Introduction

Exchange rate between two economies is the prime tool to regulate the trade flow between the economies [4]; [5]. It plays a critical role especially in developing countries, which tend to have discrepancies in external trade. A country's imports and exports of goods and services are affected by changes in the exchange rates. Mostly countries devalue their own currencies as a result of persistent and sizeable trade deficits. The logic of devaluing the domestic currency is to improve the trade balance [1]. It is clear from the figure below that the volume of imports are higher than the volume of exports in Pakistan. After 2004 there is a substantial difference in the volume of exports and imports, which provides the motivation to study the determinants of trade balance in Pakistan.



**Figure 1:** Pakistan's external trade (US dollar)

Authors of [11], [8] as well as [3] analyzed that a real depreciation of currency improves the balance of trade as well as balance of payments between host country and rest of the world. [9]; [10] discussed that weakening of local currency have ability to appreciate the trade balance. It is verified that there exists a long run impact of real effective exchange rates on trade balance, when the trade balance is used as a dependent variable [6], [12] and [2] studied the depreciation in rupee and its effect on trade balance in Pakistan. He concluded that currency depreciation is effective in improving trade balance.

Based on the discussion in the previous section it is important to understand how exchange rate changes affect the trade balance. For this purpose lag-differenced approach known as a Koyck approach is initiated. Thus, the objective of current study is to find the impact of exchange rate changes on trade balance in Pakistan by using Koyck approach to distributed lag model. The method is used to estimate the lagged and contemporaneous effect of the real exchange rate, which is not estimated in previous studies.

## 2. Data and Methodological Transformation

The annual data over the period from 1981 to 2021 is used to estimate coefficients of the regression model using Koyck approach to distributed lag model. The data set was extracted from World Development Indicators website. Theoretically, the balance of trade of an economy is dependent of real exchange rate of that economy, while other factors remaining constant. Thus, mathematically the model can be written as:

$$tb = f(r) \quad (1)$$

The data set being used for forecasting is time variant thus, the eq(1) can be written as:

$$tb_t = f(r_t) \quad (2)$$

The econometric form of eq(2) is as under:

$$\ln(tb_t) = \alpha_0 + \ln(r_t) + \varepsilon \quad (3)$$

where (tb) is representing balance of trade while (r) is used for real effective exchange rate. The focus of the study is to use distributed lag model for estimation. For this purpose [8] method of estimating distributed lag model is initiated to study the behavior of the variables.

Let consider an infinite distributed lag model

$$tb_t = \alpha + \beta_0 r_t + \beta_1 r_{t-1} + \beta_2 r_{t-2} + \dots + \mu_t \quad (4)$$

By assuming that the  $\beta$ 's are all of same sign, Koyck assumes that they decline geometrically as follows:

$$\beta_k = \beta_0 \lambda^k \quad k = 0, 1, 2, 3, \quad (5)$$

Where  $\lambda$ , such that  $0 < \lambda < 1$ , is known as the rate of decline of the distributed lag and where  $1 - \lambda$  is known as the speed of adjustment. The logic is that each successive  $\beta$  coefficient is numerically less than each preceding  $\beta$ , implying that as one goes back into the distinct past, the effect of that lag on trade balance (tb) becomes smaller. The equation 4 can suffer from sever multicollinearity problem because the close relationships between  $r_t, r_{t-1}, r_{t-2}, \dots$ . There is a geometrically declining scheme for the  $\beta$ s proposed by Koyck to avoid the multicollinearity, called the Koyck transformation. The model in equation 4 can be transformed by substituting equation 5 in equation 4

$$tb_t = \alpha + \beta_0 \lambda^0 r_t + \beta_0 \lambda^1 r_{t-1} + \beta_0 \lambda^2 r_{t-2} + \dots + \mu_t \quad (6)$$

Taking lag on both side of the equation

$$tb_{t-1} = \alpha + \beta_0 \lambda^0 r_{t-1} + \beta_0 \lambda^1 r_{t-2} + \beta_0 \lambda^2 r_{t-3} + \dots + \mu_{t-1} \quad (7)$$

Multiplying both sides by  $\lambda$

$$\lambda tb_{t-1} = \lambda \alpha + \beta_0 \lambda^1 r_{t-1} + \beta_0 \lambda^2 r_{t-2} + \beta_0 \lambda^3 r_{t-3} + \dots + \lambda \mu_{t-1} \quad (8)$$

Now subtracting equation 8 from equation 6, we get

$$tb_t = \alpha(1 - \lambda) + \beta_0 r_t + \lambda tb_{t-1} + \eta_t \quad (9)$$

Where  $\beta_0$  shows the immediate effect while  $\beta_0 / 1 - \lambda$  represents the long run impact of the model. To test the serial correlation a Durbin h test is recommended for above model instead of Durbin-Watson d test. For Koyck model the median lag – the time required for the first half of the total change in trade balance following a unit change in exchange rate, is as follows:

$$\text{Median Lag, } ML = -\log 2 / \log \lambda$$

### 3. Empirical Findings and Discussion

To measure the degree of association between exchange rate and trade balance, the researcher used Pearson correlation coefficient. The result showed that there is a 42% inverse relationship between exchange rate and trade balance which is significant at 1% level of significance. To provide the justification for autoregressive distributed lag transformation, the degree of association among exchange rate and its previous lags has been presented in table 1 below. It is evident that there is a high degree of association among different lags of exchange rate that obviously can cause multicollinearity in regression model.

**Table 1:** Covariance Analysis between Different Lags of Exchange Rate

Probability	R	R(-1)	R(-2)	R(-3)
R	1.000			
R(-1)	0.968***	1.000		
R(-2)	0.924***	0.974***	1.000	
R(-3)	0.907***	0.942***	0.980***	1.000

Note: \*\*\* shows that results are significant at 1% level of significance

The stationarity analysis of variables has been done by Augmented Dickey Fuller (ADF) test. The results as shown in table 2 revealed that trade balance is integrated at first difference while exchange rate is integrated at level.

**Table 2:** Unit Root Analysis for Level of Integration

Variable	ADF Value at Level	ADF Value at 1 <sup>st</sup> Difference	Level of Integration
Trade Balance	-1.74	-5.84***	I(1)
Exchange Rate	-4.62***	-4.01***	I(0)

ADF Critical Values: -3.61, -2.93, -2.60 at 1%, 5%, 10% respectively

The results of the regression equation are presented in the table 3. The results of estimation 1 show that there is a 152% increase in the trade balance of Pakistan if the exchange rate is kept constant. If the country depreciates its exchange rate by 1% then there is a decrease of 39% in the trade balance annually. The coefficient of Log (r) is -0.39, shows the short run impact of real effective exchange rate on the trade balance. In estimation 2, the coefficient of log (r) is -0.074 that shows the short run impact while the long run impact of the real effective exchange rate on the trade balance is -0.40. The overall significance of the model as represented by F-statistic is 46.71 and the variation in trade balance is about to 72 percent as explained by exchange rate.

**Table 3:** Estimated Results for Koyck Model when Trade Balance is Dependent Variable

	Estimation 1	Estimation 2
Variables	Coefficients	Coefficients
constant	1.52**	0.288
Log (r)	-0.39***	-0.074
Log (tb(-1))		0.817***
	F-Statistic = 7.98	F-Statistic = 46.71
	R-Square = 0.173	R-Square = 0.722

Note: \*\*, \*\*\* shows the significance at 5% and 1% level of significance

In the table 3 the coefficient of log lagged dependent variable is 0.817 which shows the rate of decay of the distributed lag and the speed of adjustment is only 18% annually. Also it confirms that as the rate of decline is high the speed of adjustment is slow. As the value of  $\lambda = 0.817$  the value of the median lag is 3.43 which shows that 50% of the total change in the trade balance is accomplished in more than 3 periods. The value of Durbin h is approximately equal to 5.0265 which is about the bench mark. Also the value of the Durbin Watson d is greater than the value of the R square, thus the regression is not spurious.

#### 4. Conclusion

The prime objective of this research was to initiate an autoregressive distributed lag approach to analyze the impact of exchange rate variation on trade balance when there can be a possible chance of existence of multicollinearity among the explanatory variables. To cater this problem Koyck transformation has been used and then regression analysis was used to find the immediate and long term relationship between exchange rate and trade balance. The analysis was made on annual data ranging from 1981 to 2021. The results revealed that there exist an inverse relationship between the trade balance and the real effective exchange rates and reduction in the exchange rate will bring the improvement in the trade balance in the short run as well as the long run. It is also verified with the help of data that distributed lag transformation is effective in combating multicollinearity.

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MA: initiated the concept, estimated statistical results and written the manuscript  
 SH: provided assistance in mathematical calculation and proof read the manuscript

#### Competing Interests

The author(s) hereby declare that they have no competing interests.

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