IJEMD-M, 1 (3) (2022) 106 - 117 https://doi.org/10.54938/ijemdm.2022.01.3.122

International Journal of Emerging Multidisciplinaries: Mathematics

Research Paper
Journal Homepage: www.ojs.ijemd.com
ISSN: 2790-1998 (print), 2790-3257 (online)

Three Dimensional MHD Viscous Flow Under the Influence of Thermal Radiation and Viscous Dissipation

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Abstract

The present study elucidates the results on the mathematical modeling and numerical study for the viscous flow demeanor past over the plane horizontal surface stretched nonlinearly in two sideways. Furthermore, a comprehensive analysis on the effects of magnetic field, thermal radiation and viscous dissipation are considered and observed. Cartesian coordinate system is employed for modelling the flow equations. In this research water act as a traditional thermal fluid. Three distinct nanoparticles namely Gold (Au), Aluminum (Al) and Silver (Ag) are suspended. Numerical and analytical solution for the resulting differential equations demonstrates the flow demeanor for velocity and temperature distribution are transformed into ordinary differential equations by aiding the suitable transformations. The accuracy of exact solution is also validated by using shooting method in MATHEMATICA 10.0. The impacts of multifarious parameters particularly Eckert number, Prandtl number, Magnetic parameter, and heat source parameter for the suspended nanoparticles on velocity and temperature are given through graphs.

Keywords: MHD; Thermal radiation; viscous dissipation; Nanoparticles.

1. Introduction

For several years, in the numerous engineering applications fluids having low thermal conductivity is a serious matter in heat transmission processes. Owing to smaller molecular size and enormous thermal features, these nanometers sized (1-100nm) shows peculiarities among the physical and chemical demeanor. By suspending the small number of nanoparticles (less than 1%), thermal conductivity of traditional thermal fluid is enhanced. As a consequence, heat transfer characteristic enhanced by twice. These nanoparticles depict the smoothness in their flow without clogging in micro channel and act as a liquid molecule. Nano fluids depicts great diversities in terms

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of extensive functional applications in biomedical and scientific fields including electronic devices, heating and boiling process of energy, radiators, nuclear reactors and lots of other. In recent times, analysis proposed that the base fluid manifest diversity in characteristics after reduce to minimum nanoparticles. Consequently, thermal proficiency of base fluids such as (Ethylene, Oil, Water and Glycol) is intensified with the inclusion of such nanoparticles. In 1995, the Nano fluid idea was initially confered by Choi [1] and Choi et al. [2]. Heat transport phenomena was studied by Kakac et al. and Xuan et al. [3, 4]. Potential application of Nano fluid was examined by Wong et al. [5]. Heat convection of Nano fluid was examined by Buongiorno [6]. Cho [7] inspected the hydrodynamic and temperature distribution features under consideration of metal oxides particle. Kuznetrove and Neiled [8] conducted an analysis for the convective heat transmission phenomenon in a Nano fluid configuring the vertical plate. Oudina et al. [9] discussed the convective heat transmission of MgO-Ag/ water magneto-hybrid Nano fluid flow demeanor configured by a specific porous enclosure. Naveed et al. [10] has given a unique idea by investigating the Thermophoresis and Brownian motion impacts in consideration of Nano fluid flow past over a curved geometry. Analytical approach for the Nano fluid configured by stretched surface was given by Hassani et al. [11]. Sheikholeslami and Abelman [12] work based on dual phase Nano fluid flow for the heat transport configured in an annulus. Wakif et al. [13] examined the magneto-convection for the Nano fluid namely alumina-water among the thin horizontal layers with the aid of generalized updated Buongiorno's Model. Rasool et al. [14] discussed the chemical reaction characteristic and convective limiting conditions for the Powell-Eyring Nano fluidic flow behavior past over a radiative Riga plate.

Magnetic field also depicts vast impact Ness in both natural and industrial processes in terms of fluids and flow demeanor. Earth magnetic field functioning as a shield facing the fatal radiations. Sun magnetic field considered to be source of production for sunspots and Sun flares. We entitled all above phenomena by a single word as Magneto hydrodynamics [MHD]. In 2021, Sohail et al. [15] devised bio convection mechanism taking into account MHD Carreau liquid for the boundary layer flow past over a heated disk. Also, in the same year M. Sohail [16] investigated the non-Newtonian fluid and observed the radiative flow demeanor of MHD under the influence of Joule heating by embodying the modified version of heat flux model. Numerical calculations for oscillatory Magneto hydrodynamics configured by cylindrical annulus was presented by Oudina et al. [17] and discussed the Prandtl number impacts. The prime focus of Abbas et al. [18] analysis is on the study of impacts of thermal radiations over the MHD slip flow demeanor for Nano fluid towards a curved stretching surface. Rosca and Pop [19] studied the Magneto hydrodynamic flow behavior past over a penetrable stretching surface. Shahid et al. [20] proposed the numerical approach of stimulation energy on Magneto hydrodynamic Nano fluid flow behavior. Waqas and Wakif [21] proposed the activation energy and magnetic field significance on the attributes of stratified mixed radiative convection couple-stress Nano fluid flow demeanor within motile organisms.

Thermal radiations play an extensive and crucial part and are vital in view of their significance in various industrial procedures like glass manufacturing as well as heating system scheme. Moreover, Cosmo technology applied for instance aeronautical aerodynamics rockets. Abdelmalek et al. [22] considered the Williamson Nano fluid containing microorganism and observed the radiative flow behavior under the influence of heat source and activation energy. A numerical analysis was examined by Rasool et al. [23] for the Marangoni forced convective flow demeanor of Casson Nano fluid flow behavior under the influence of Lorentz forces produced by Riga plate. Furthermore in 2019, Rasool et al. [24] suggested a numerical approach for the marangoni convective flow demeanor under the influence of thermal radiations and Lorentz force along with chemical reaction. Benos et al. [25] proposed the numerical analysis under the influence of radiations, the Navier slip limiting condition along with heat source and Lorentz force. MHD for viscid Nano fluid accompanied with radiative impacts was investigated by Hussain et al. [26]. Imtiaz et al. [27] work rely on the study of radiation impacts on the carbon nanotubes movement past over a stretching rotary disk. The quality of final product is mutually aligned with the rate of heat transport towards the stretching surface. Many researchers have kept an eye on this peculiar field of study keeping in mind the essential requisite features in terms of the industrial procedures. Fundamental flow

demeanor configured by stretching surface was first studied by Crane [28]. The idea was presented and discussed by Siddappa et al. [29] for the flow demeanor towards the stretching surface. Puneeth et al. [30], Abel et al. [31] and Sharidan et al. [32] examined the fluid behavior towards a stretching sheet.
Viscous dissipation occurs in geological processes, massive gasses in space as well as in strong gravitational fields and bigger plants. Furthermore, viscous dissipation provides additional heat in the flow mechanism. Nayak et al. [33] investigated the viscous dissipation impacts on magneto hydrodynamic flow demeanor for the viscoelastic fluid's past over an inclined surface. Viscous dissipation being directly related to Nano fluid viscosity, considered to be the function of concentration consequently related to the temperature profile. Sohail et al. [34] work based the three-dimensional flow behavior and observed the joule heating and viscous dissipation impacts for the Casson fluid model incorporating temperature reliant conductance with the aid of numerical approach namely shooting method. Abbasi et al. [35] examined the ameliorated thermal efficiency along with optimized approach for the Copper Aluminum oxide under the influence of viscous dissipation and Joule heating.

2. Modeling and its physical description

Consider the three-dimensional incompressible flow demeanor configured by a plane plastic sheet. It is assumed that sheet is located at z=0 where u, v and w is considered to be velocity components in the x-axes, y-axes and z- axes.
The continuity, momentum and energy equation for the Nano fluid in the presence of magnetic field. Thermal radiations and viscous dissipation are given by Eqs. (1), (2), (3) and (4) as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1}$$

For x-component of velocity

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=\nu_{nf}\frac{\partial^2 u}{\partial^2 z}-\frac{\sigma_{nf}B_o^2}{\rho_{nf}}.u. \tag{2}$$

For y-component of velocity

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=\nu_{nf}\frac{\partial^2 v}{\partial^2 z}-\frac{\sigma_{nf}B_o^2}{\rho_{nf}}v. \tag{3}$$

The Energy equation given by

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\alpha_{nf}\frac{\partial^2 T}{\partial^2 z}-\frac{1}{(\rho C_p)_{nf}}\frac{\partial q_r}{\partial z}+\frac{\mu_{nf}}{(\rho C_p)_{nf}}\left[\left(\frac{\partial u}{\partial z}\right)^2+\left(\frac{\partial v}{\partial z}\right)^2\right] \tag{4}$$

Where $\frac{\sigma_{nf}B_o^2}{\rho_{nf}}.u$ and $\frac{\sigma_{nf}B_o^2}{\rho_{nf}}.v$ represents the MHD term along x and y components of velocity, respectively while $\frac{1}{(\rho C_p)_{nf}}\frac{\partial q_r}{\partial z}$ represents thermal radiation term. Furthermore, $\frac{\mu_{nf}}{(\rho C_p)_{nf}}\left[\left(\frac{\partial u}{\partial z}\right)^2+\left(\frac{\partial v}{\partial z}\right)^2\right]$ represents viscous dissipation term.

The Corresponding proposed boundary conditions is

$$u=U_o=c(x+y)^m, v=V_o=d(x+y)^m, w=0, \quad T=T_o=T_\infty+C(x+y)^m, \quad At \quad z=0, \tag{6}$$

$$u = 0, v = 0, T \to T_\infty \qquad As \ z \to \infty$$

Mathematical expression for distinct thermo physical attributes associated with Nano fluid are represented by

$$\frac{\mu_{nf}}{\mu_f} = 1 + 4.93\gamma + 222.4\gamma^2.$$
$$\frac{K_{nf}}{K_f} = 1 + 2.944\gamma + 19.672\gamma^2.$$
$$\frac{\rho_{nf}}{\rho_f} = (1-\gamma) + \gamma\left(\frac{\rho_s}{\rho_f}\right).$$
$$\frac{(\rho C_p)_{nf}}{(\rho C_p)_f} = 1 - \gamma + \gamma \ \frac{(\rho C_p)_s}{(\rho C_p)_f}. \tag{4}$$
$$\frac{\sigma_{nf}}{\sigma_f} = 1 + \frac{3\left(\frac{\sigma_s}{\sigma_f} - 1\right)\gamma}{2 + \frac{\sigma_s}{\sigma_f} - \left(\frac{\sigma_s}{\sigma_f} - 1\right)\gamma}.$$

Where γ denotes the nanoparticle volume fraction.

Here, $\mu_{nf}, \sigma_{nf}, K_{nf}, (\rho C_p)_{nf}$ and ρ_{nf} epresents dynamic viscosity, electrical conductivity, Thermal conductivity, heat capacity and density for Nano fluid. Moreover, γ represents the nanoparticle volume fraction and in the subscript f and s referred to as traditional thermal fluid [water] and solid nanoparticles namely Aluminum [Al], Silver [Ag] and Gold [Au] respectively as mentioned in table 1 and table 2 with their mathematical relation. For similar solution we have introduced the following similarity transformation of the form given by:

$$u = a(x+y)^m f'(\xi), v = b(x+y)^m g'(\xi),$$
$$w = -\sqrt{av}(x+y)^{\frac{m-1}{2}}\left(\frac{m+1}{2}(f(\xi)+g(\xi)) + \frac{m-1}{2}\xi(f'(\xi)+g'(\xi))\right).$$
$$\theta = \frac{T-T_\infty}{T_o - T_\infty}, \qquad \xi = \sqrt{\frac{a}{v}}(x+y)^{\frac{m-1}{2}} z.$$

Equation (1) is identically satisfied. Moreover, Transformed form of equations (2), (3), (4) and (5) are given as follows:

$$\frac{\frac{\mu_{nf}}{\mu_f}}{\frac{\rho_{nf}}{\rho_f}} f''' - \frac{\frac{\sigma_{nf}}{\sigma_f}}{\frac{\rho_{nf}}{\rho_f}} Mf' - mf'(f' + g') \quad + \frac{(m+1)}{2} f''(f + g) = 0. \tag{6}$$

$$\frac{\frac{\mu_{nf}}{\mu_f}}{\frac{\rho_{nf}}{\rho_f}} g''' - \frac{\frac{\sigma_{nf}}{\sigma_f}}{\frac{\rho_{nf}}{\rho_f}} Mg' - m\,g'(f' + g') \quad + \frac{(m+1)}{2} g''(f + g) = 0. \tag{7}$$

$$\frac{\frac{K_{nf}}{K_f}}{\frac{(\rho C_p)_{nf}}{(\rho C_p)_f}}\frac{1}{Pr}(1+Nr)\theta''+\frac{\frac{\mu_{nf}}{\mu_f}}{\frac{(\rho C_p)_{nf}}{(\rho C_p)_f}}Ec\left(f''^2+g''^2\right)-m\theta(f'+g')+\frac{(m+1)}{2}(f+g)\theta'=0 \quad (8)$$

$$f(0)=g(0)=0, \quad f'(0)=1, g'(0)=\lambda, \ \theta(0)=1, \quad f'(\infty)\to 0, g'(\infty)\to 0, \theta(0)\to 0, \quad (9)$$

Table 1. Thermal and physical attributes for Nano fluid configuration.

Attributes	$\rho\left(\frac{kg}{m^3}\right)$	$C_p\left(\frac{J}{kgK}\right)$	$k\left(\frac{W}{mK}\right)$	$\sigma\left(\frac{S}{m}\right)$
Water, f	997.1	4179	0.613	5.5×10^{-6}
Silver, s	10,800	234	429	66.7×10^{6}
Gold, s	19300	129	310	49.0×10^{6}
Aluminum, s	3970	765	40	35×10^{6}

Table 2: Relationship of Physical Quantities.

FOR GOLD (Au),s When $\gamma=0.01$	$\frac{\mu_{nf}}{\mu_f}=1.07154$	$\frac{K_{nf}}{K_f}=1.03140$	$\frac{\rho_{nf}}{\rho_f}=1.18356$	$\frac{(\rho C_p)_{nf}}{(\rho C_p)_f}=0.99594$	$\frac{\sigma_{nf}}{\sigma_f}=1.03030$
FOR SILVER (Ag),s When $\gamma=0.01$	$\frac{\mu_{nf}}{\mu_f}=1.07154$	$\frac{K_{nf}}{K_f}=1.03140$	$\frac{\rho_{nf}}{\rho_f}=1.09831$	$\frac{(\rho C_p)_{nf}}{(\rho C_p)_f}=0.99606$	$\frac{\sigma_{nf}}{\sigma_f}=1.03030$
FOR ALUMINIUM(Al),s When $\gamma=0.01$	$\frac{\mu_{nf}}{\mu_f}=1.07154$	$\frac{K_{nf}}{K_f}=1.03140$	$\frac{\rho_{nf}}{\rho_f}=1.02981$	$\frac{(\rho C_p)_{nf}}{(\rho C_p)_f}=0.99072$	$\frac{\sigma_{nf}}{\sigma_f}=1.03030$

3. Solution Methodology

In mathematical modeling the fluid flow problem leads to the form of differential equations. These differential equations in numerous cases are highly non-linear and obtaining their results is not a simple task. Different numerical and analytical procedures for the solutions of these equations have been used. We have utilized shooting method with Runge-Kutta of fourth order algorithm to resolve these equations.

4. Shooting method

Shooting method is an iterative method which is very popular for two point's boundary value problems. In this method boundary value problem of higher order is first reduced to system of first order initial value problem by supposing a missing condition. Then the solution of initial value problem instead of known boundary value problem will be obtained. Runge-Kutta method of order four is applied to solve the initial value problem.

5. Result and discussion

Currently, in this study to solve the momentum equation (2, 3), energy equation (4) along with imposed boundary conditions given in equation (5), numerically we have employed R.K fourth order based Shooting Method and optimal homotopy analysis method (OHAM). A comparative analysis for diverse nanoparticles namely Silver (Ag), Gold (Au) and Aluminum (Al) for the velocity distribution and temperature distribution is carried out. The graphical results of dissimilar involved parameters are evaluated and presented through Figs. 1-11.

Figs. 1-3 are plotted to display the influence of different parameters specifically magnetic parameter (M), stretching rates ratio (λ) and power law index (m) on the velocity field along x-axes. It shows that velocity profile declined with the increasing values of parameters under the comparative analysis of proposed nanoparticles. Similarly, Velocity field along y-axes under the influence of Magnetic parameter (M), stretching rates ratio (λ) and power law index (m) is shown in Figs. 4-6. With the growing values of power law index (m) and magnetic parameter (M), velocity field is decreased whilst for stretching rates ratio (λ) it is intensified. Fig. 7 depicts the influence of Eckert number Ec on the temperature field. It is observed that for distinct nanoparticles with upshot values of Eckert number, temperature field is enhanced. It is found in Figs. 8-10 that temperature profile is significantly affected by the comparative analysis of proposed nanoparticles $(Ag, Au\ and\ Al)$ and shows decaying behavior with the increasing values of Pr, $\lambda\ and\ m.$ Figure 11 made to manifest the radiation parameter Nr impacts and higher values of radiation parameter ameliorate the conduction effects, consequently temperature field is enhanced under the comparative analysis of the proposed nanoparticles specifically Silver (Ag), Gold (Au) and Aluminum (Al).

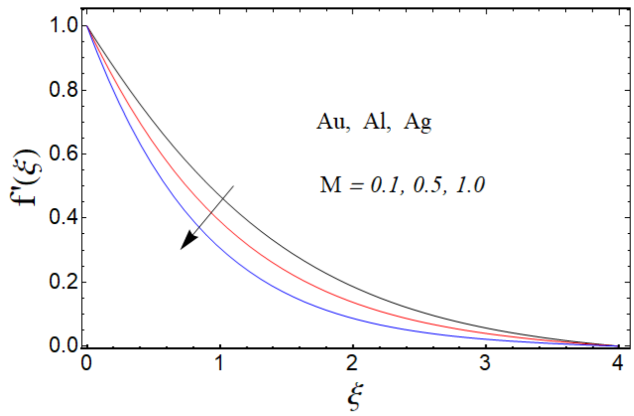

Figure 1. Comparison of three nanoparticles namely Gold (Au), Aluminum (Al) and Silver (Ag) for the various values of $M = 0.1, 0.5, 1.0$ for the velocity field along x-component.

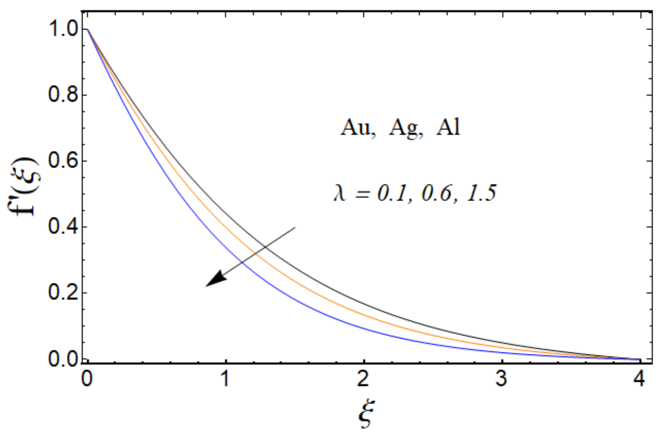

Figure 2. Comparison of three nanoparticles namely Gold (Au), Silver (Ag) and Aluminum (Al) for the various values of $\lambda = 0.1, 0.6, 1.5$ for the velocity field along x-component

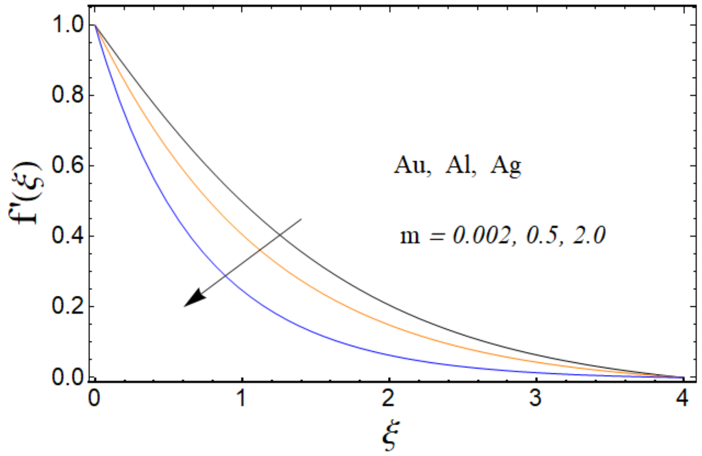

Figure 3. Correlation of three nanoparticles namely Gold (Au), Aluminum (Al) and Silver (Ag) for the distinct values of $m = 0.002, 0.5, 2.0$ for the velocity field along x-component.

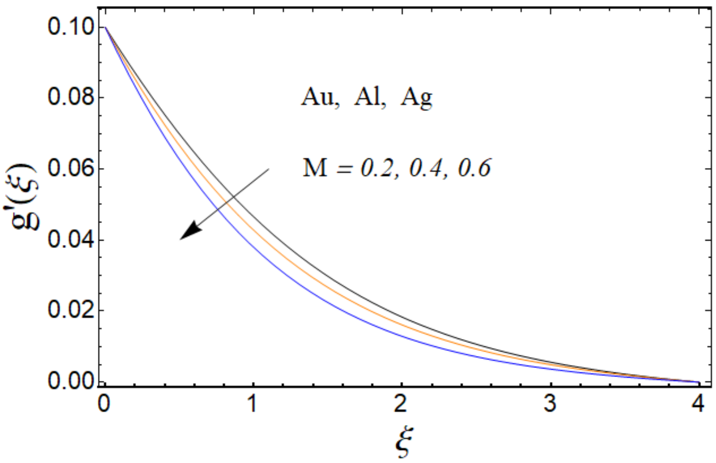

Figure 4. Correlation of three nanoparticles namely Gold (Au), Aluminum (Al) and Silver (Ag) for the various values of $M = 0.2, 0.4, 0.6$ for the velocity field along y-component.

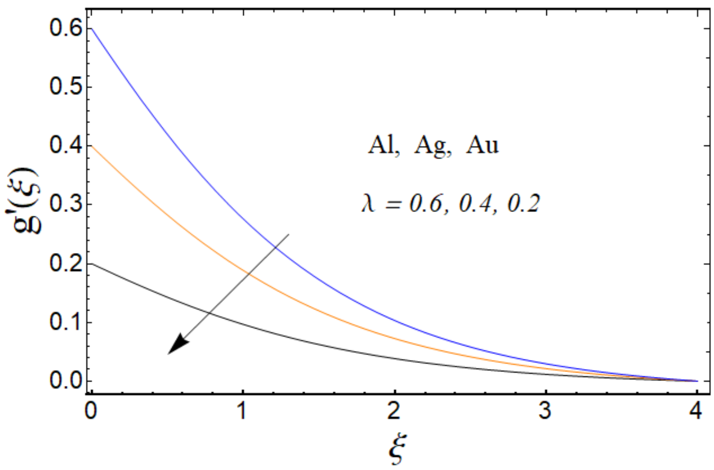

Figure 5. Correlation of three nanoparticles namely Aluminum (Al), Silver (Ag) and Gold (Au) for the different values of $\lambda = 0.6, 0.4, 0.2$ for the velocity field along y-component.

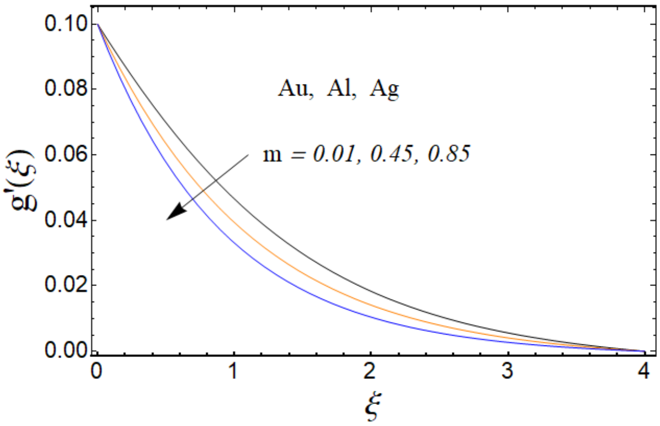

Figure 6. Correlation of three nanoparticles namely Aluminum (Al), Gold (Au) and Silver (Ag) for the different values of $\lambda = 0.01, 0.45, 0.85$ for the velocity field along y-component.

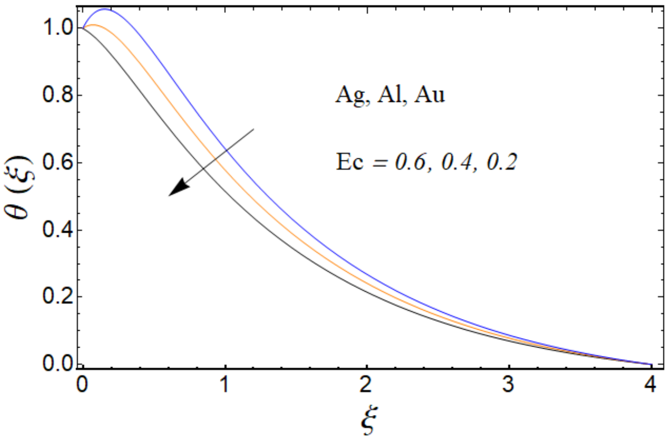

Figure 7. Correlation of three nanoparticles namely Silver (Ag), Aluminum (Al) and Gold (Au), for the different values of $Ec = 0.6, 0.4, 0.2$ for the temperature field.

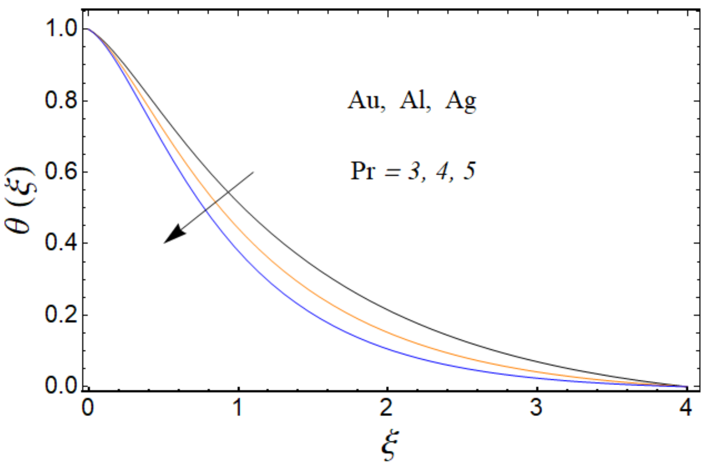

Figure 8. Deviation in the temperature profile for distinct values of $\mathrm{Pr} = 3, 4, 5$ for nanoparticles gold (Au), Aluminum (Al), and Silver (Ag) respectively.

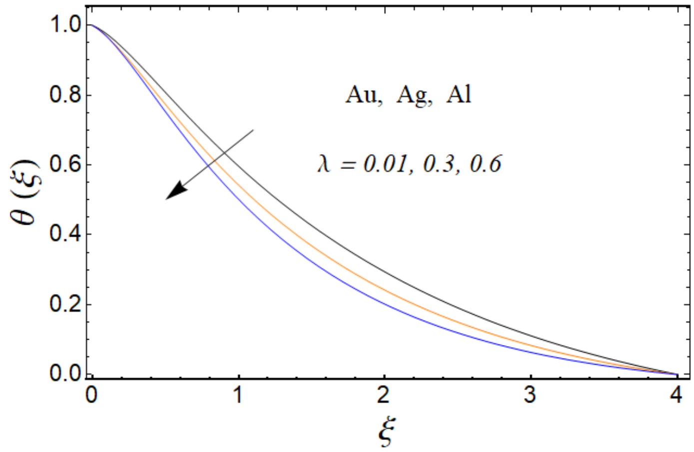

Figure 9. Comparison of three nanoparticles namely Gold (Au), Silver (Ag) and Aluminum (Al) for the diverse values of $\lambda = 0.01, 0.3, 0.6$ for the temperature field.

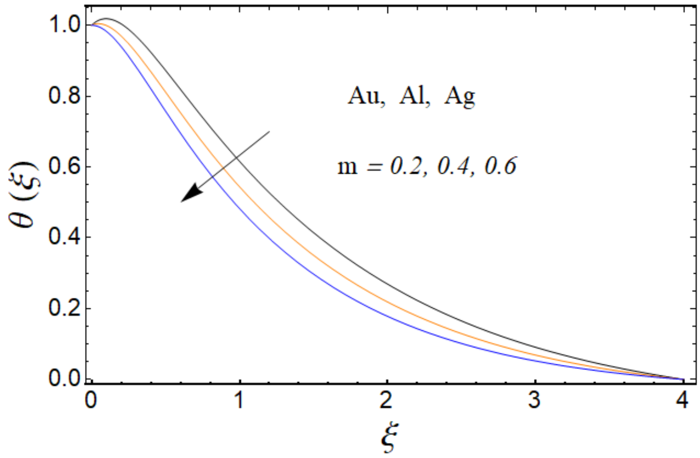

Figure 10. Comparison of three nanoparticles namely Gold (Au), Aluminum (Al) and Silver (Ag) for the diverse values of $m = 0.2, 0.4, 0.6$ for the temperature field.

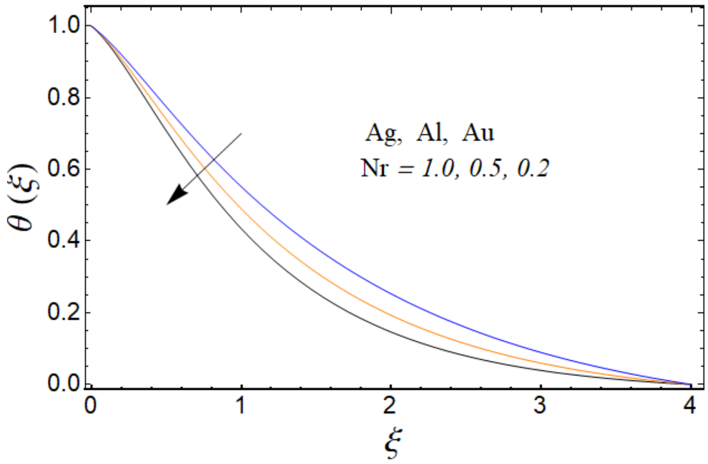

Figure 11. Comparison of three nanoparticles namely Silver (Ag), Aluminum (Al) and Gold (Au) for the diverse values of $Nr = 1.0, 0.5, 0.2$ for the temperature field.

6. Conclusions

The prime focus of current study was to present the meaningful investigation for the viscous flow past over a stretching sheet. The optimization of energy is carried out by taking into account thermal radiations and viscous dissipation.

Summarized key points of current study are as follows

- Velocity field along $x-axes$ shows decreasing demeanor for higher values of M, $\lambda\, and\, m$.
- Velocity field along $y-axes$ is enhanced for upshot values of λ while it shows opposite demeanor for varying in Magnetic parameter (M) and power law index (m).
- For growing values of Ec and Nr temperature field is inclined whilst it is declined for Pr, $\lambda\, and\, m$.

Availability of data and material

The datasets used and/or analyzed during the current study available from the corresponding author on reasonable request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References

[1] Choi, S. U., & Eastman, J. A. *Enhancing thermal conductivity of fluids with nanoparticles* (No. ANL/MSD/CP-84938; CONF-951135-29). Argonne National Lab.(ANL), Argonne, IL (United States), 1995.

[2] Choi, S. U. S., Zhang, Z. G., Yu, W., Lockwood, F. E., & Grulke, E. A. Anomalous thermal conductivity enhancement in nanotube suspensions. *Applied physics letters*, **79**(14), 2252-2254 (2001).

[3] Kakaç, S., & Pramuanjaroenkij, A. Review of convective heat transfer enhancement with nanofluids. *International journal of heat and mass transfer*, **52**(13-14), 3187-3196 (2009).

[4] Xuan, Y., & Li, Q. Heat transfer enhancement of nanofluids. *International Journal of heat and fluid flow*, **21**(1), 58-64 (2000).

[5] Wong, K. V., & De Leon, O. Applications of nanofluids: current and future. *Nanotechnology and Energy*, 105-132 (2017).

[6] Buongiorno, J. Convective transport in nanofluids, 2006.

[7] Pak, B. C., & Cho, Y. I. Hydrodynamic and heat transfer study of dispersed fluids with submicron metallic oxide particles. *Experimental Heat Transfer an International Journal*, **11**(2), 151-170 (1998).

[8] Kuznetsov, A. V., & Nield, D. A. Natural convective boundary-layer flow of a nanofluid past a vertical plate. *International Journal of Thermal Sciences*, **49**(2), 243-247 (2010).

[9] Mebarek-Oudina, F., Redouane, F., & Rajashekhar, C. Convection heat transfer of MgO-Ag/water magneto-hybrid nanoliquid flow into a special porous enclosure. *Alger. J. Renew. Energy Sustain. Dev*, **2**(2), 84-95 (2020).

[10] Naveed, M., Abbas, Z., & Sajid, M. Thermophoresis and Brownian effects on the Blasius flow of a nanofluid due to a curved surface with thermal radiation. *The European Physical Journal Plus*, **131**(6), 1-9 (2016).

[11] Hassani, M., Tabar, M. M., Nemati, H., Domairry, G., & Noori, F. An analytical solution for boundary layer flow of a nanofluid past a stretching sheet. *International Journal of Thermal Sciences*, **50**(11), 2256-2263 (2011).

[12] Sheikholeslami, M., & Abelman, S. Two-phase simulation of nanofluid flow and heat transfer in an annulus in the presence of an axial magnetic field. *IEEE Transactions on Nanotechnology*, **14**(3), 561-569 (2015).

[13] Wakif, A., Boulahia, Z., Amine, A., Animasaun, I. L., Afridi, M. I., Qasimd, M., & Sehaqui, R. Magneto-convection of alumina-water nanofluid within thin horizontal layers using the revised generalized Buongiorno's model. *Frontiers in Heat and Mass Transfer (FHMT)*, **12**, (2018).

[14] Rasool, G., & Zhang, T. Characteristics of chemical reaction and convective boundary conditions in Powell-Eyring nanofluid flow along a radiative Riga plate. *Heliyon*, **5**(4), e01479 (2019).

[15] Sohail, M., Nazir, U., Chu, Y. M., Al-Kouz, W., & Thounthong, P. Bioconvection phenomenon for the boundary layer flow of magnetohydrodynamic Carreau liquid over a heated disk. *Scientia Iranica*, **28**(3), 1896-1907 (2021).

[16] Sohail, M., Ali, U., Zohra, F. T., Al-Kouz, W., Chu, Y. M., & Thounthong, P. Utilization of updated version of heat flux model for the radiative flow of a non-Newtonian material under Joule heating: OHAM application. *Open Physics*, **19**(1), 100-110 (2021).

[17] Mebarek-Oudina, F., & Makinde, O. D. Numerical simulation of oscillatory MHD natural convection in cylindrical annulus: Prandtl number effect. In *Defect and Diffusion Forum*. Trans Tech Publications Ltd. **387**, 417-427 (2018).

[18] Abbas, Z., Naveed, M., & Sajid, M. Hydromagnetic slip flow of nanofluid over a curved stretching surface with heat generation and thermal radiation. *Journal of Molecular Liquids*, **215**, 756-762 (2016).

[19] Roşca, N. C., & Pop, I. Unsteady boundary layer flow over a permeable curved stretching/shrinking surface. *European Journal of Mechanics-B/Fluids*, **51**, 61-67 (2015).

[20] Shahid, A., Huang, H. L., Khalique, C. M., & Bhatti, M. M. Numerical analysis of activation energy on MHD nanofluid flow with exponential temperature-dependent viscosity past a porous plate. *Journal of Thermal Analysis and Calorimetry*, **143**(3), 2585-2596 (2021).

[21] Waqas, H., Wakif, A., Al-Mdallal, Q., Zaydan, M., Farooq, U., & Hussain, M. Significance of magnetic field and activation energy on the features of stratified mixed radiative-convective couple-stress nanofluid flows with motile microorganisms. *Alexandria Engineering Journal*, **61**(2), 1425-1436 (2022).

[22] Abdelmalek, Z., Khan, S. U., Waqas, H., Riaz, A., Khan, I. A., & Tlili, I. A mathematical model for bioconvection flow of Williamson nanofluid over a stretching cylinder featuring variable thermal conductivity, activation energy and second-order slip. *Journal of Thermal Analysis and Calorimetry*, **144**(1), 205-217 (2021).

[23] Rasool, G., Shafiq, A., & Khalique, C. M. Marangoni forced convective Casson type nanofluid flow in the presence of Lorentz force generated by Riga plate. *Discrete & Continuous Dynamical Systems-S*, **14**(7), 2517 (2021).

[24] Rasool, G., Zhang, T., Shafiq, A., & Durur, H. Influence of chemical reaction on Marangoni convective flow of nanoliquid in the presence of Lorentz forces and thermal radiation: A numerical investigation. *J. Adv. Nanotechnol*, **1**(1), 32-49 (2019).

[25] Benos, L. T., Mahabaleshwar, U. S., Sakanaka, P. H., & Sarris, I. E. Thermal analysis of the unsteady sheet stretching subject to slip and magnetohydrodynamic effects. *Thermal Science and Engineering Progress*, **13**, 100367 (2019).

[26] Hussain, A., Sarwar, L., Akbar, S., Malik, M. Y., & Ghafoor, S. Model for MHD viscoelastic nanofluid flow with prominence effects of radiation. *Heat Transfer—Asian Research*, **48**(2), 463-482 (2019).

[27] Imtiaz, M., Hayat, T., Alsaedi, A., & Ahmad, B. Convective flow of carbon nanotubes between rotating stretchable disks with thermal radiation effects. *International journal of heat and mass transfer*, **101**, 948-957 (2016).

[28] Crane, L. J. Flow past a stretching plate. *Zeitschrift für angewandte Mathematik und Physik ZAMP*, **21**(4), 645-647 (1970).

[29] Siddappa, B., & Hiremath, P. S. Rivlin-Ericksen fluid flow past a stretching plate with suction. *Kyungpook Mathematical Journal*, **20**(2), 267-272 (1980).

[30] Puneeth, V., Manjunatha, S., Gireesha, B. J., & Gorla, R. S. R. Magneto convective flow of Casson nanofluid due to Stefan blowing in the presence of bio-active mixers. *Proceedings of the Institution of Mechanical Engineers, Part N: Journal of Nanomaterials, Nanoengineering and Nanosystems*, **235**(3-4), 83-95 (2021).

[31] Abel, M. S., Khan, S. K., & Prasad, K. V. Study of visco-elastic fluid flow and heat transfer over a stretching sheet with variable viscosity. *International journal of non-linear mechanics*, **37**(1), 81-88 (2002).

[32] Sharidan, S., Mahmood, M., & Pop, I. Similarity solutions for the unsteady boundary layer flow and heat transfer due to a stretching sheet. *Applied Mechanics and Engineering*, **11**(3), 647 (2006).

[33] Nayak, M. K., Dash, G. C., & Singh, L. P. Unsteady radiative MHD free convective flow and mass transfer of a viscoelastic fluid past an inclined porous plate. *Arabian Journal for Science and Engineering*, **40**(11), 3029-3039 (2015).

[34] Sohail, M., Chu, Y. M., El-Zahar, E. R., Nazir, U., & Naseem, T. Contribution of joule heating and viscous dissipation on three dimensional flow of Casson model comprising temperature dependent conductance utilizing shooting method. *Physica Scripta*, **96**(8), 085208 (2021).

[35] Abbasi, A., Khan, S. U., Farooq, W., & Khan, M. I. Optimized analysis and enhanced thermal efficiency of copper–aluminum oxide nanoparticles under the influence of Joule heating and viscous dissipation. *The European Physical Journal Plus*, **136**(10), 1026 (2021).