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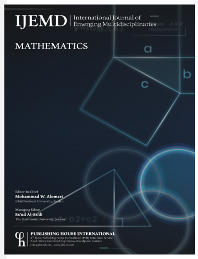

Heat and Mass Transfer of Viscous Fluid in a Permeable Channel with Reabsorbing Walls

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Abstract

An analytical investigation is made to determine the heat and mass transfer mechanism of non-isothermal highly viscous fluid in a long narrow porous channel. The walls of the channel are maintained at the same temperature. The mathematical model is developed by using the continuity, momentum, energy and diffusion equations. Analytical solutions are establish to get the expressions of velocity field, pressure distribution, mass flow rate, wall shear stress, temperature profile, mass concentration distribution as well as the heat transfer rate (Nusselt number) and mass transfer rate (Sherwood number) with involved physical parameters. Numerical results are graphically sketched to describe the role of different parameters involved in the governing equations. It is observed that longitudinal velocity component is higher at the entrance region as compared to middle position and exit region of the channel. Temperature field decreases at the centerline of the channel with increasing suction (reabsorption) parameter. Concentration decreases from surface to the center of the channel and with increasing suction parameter concentration increases. Heat transfer rate decreases while mass transfer rate increases along the channel. With increasing Brinkman number heat transfer increases and its strength at the entrance region is higher than exit region, while opposite behaviour is observed for mass transfer inside the channel.

Keywords: *Permeable channel; analytic solution; Brinkman number; heat transfer and mass transfer rates.*

1. Introduction

The occurrence of laminar flow in ducts (tube or channel) with permeable walls gained much attention during passed years due to its numerous applications in bioengineering, for example in the flow of blood in dialyzer (artificial kidney) [1], glomerular filtrate in proximal convoluted tubule [2], blood in membrane oxygenators [3]. Moreover, its applications are found in transpiration cooling boundary layer control [4] as well as in the design of filters [5].

The pioneer work of steady, laminar flow of linearly viscous (Newtonian) fluid in porous channel can be seen in Bermans paper [6]. He reduced the Navier Stokes equations into ordinary differential equation using a similarity transformation and obtained approximate solution for small Reynolds number on the basis of uniform suction across the walls. His work was extended by Sellars [7], and obtained series solution for large Reynolds number. Later on, approximate solution of flow of laminar fluid flow in permeable channel with uniform suction/injection was obtained by Yuan [8], who investigated the flow in case of moderate to high suction or injection velocities across the walls. Terrill et al. [9] have studied the flow through permeable channel with different permeability for small Reynolds number to get the velocity components and pressure drop. The velocity profile for steady, creeping flows with variable wall suction for both channel and tube flow was obtained by Kozinski et al [10]. They assumed an exponential decrease in wall permeation rate with axial distance as the boundary condition for the normal velocity at the porous walls. In the last few years, Muthu et al. [11] investigated the flow of viscous fluid in a porous channel, assuming that the bulk flow

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decreases with axial distance of the channel. They obtained approximate solutions for velocity components and discussed the flow variations through graphs at different positions. They further extended their work to non-uniform channel [12, 14] with slip condition at the walls. Recently, Siddiqui et al. [15] extended the work of Kozinski et al. [10] to uniform wall reabsorption (suction). They developed a mathematical model to study the flow of viscous fluid flow in a permeable channel with uniform suction across the walls. In the previous work [6-15], the hydrodynamics of viscous fluid flow in a permeable channel with uniform as well as variable suction/injection has been studied.

Heat and mass transfer mechanism has several applications in biosciences and in industries, for example, the air movement in the pores of fibrous material due to heat convection, conduction in tissues and radiation between environment and its surface, food processing mass and diffusion in reverse osmosis. In 1965, Terrill presented the analytical solution of the two dimensional laminar and non-isothermal flow of linearly viscous fluid in a porous channel [16]. In 1968, he extended his work and obtained the expression of temperature profile and mass concentration [17]. Later, Walker et al. presented the temperature field for viscous fluid flow in porous channels with mass transfer through the walls of the channel [18]. Raithby [19] found the exact solution of the diffusion equation, while numerical solutions of the energy equation were discussed for small injection rates. Mahmud et al. [20] used the similarity solutions to evaluate the pressure gradient, Nusselt number, and Sherwood number in the two dimensional flow in a permeable channel. Singh [21] also have studied the heat and mass transfer of Newtonian fluid flow over a heated vertical flat porous surface with heat source/sink. In recent years, investigations of heat and mass transfer in peristalsis have been reflected by few researchers [22-25]. They obtained the mathematical expressions for velocity components and pressure gradient using long-wavelength and small Reynolds number approximations.

Based on the above discussion, it is deduced that the study of heat and mass transfer in a porous channel is important due to its application in bioscience and industries. In the previous work [10-15], the mathematical study of heat and mass transfer has not been cited so far. Therefore, this paper will provide the heat and mass transfer in a porous channel with uniform wall suction (reabsorption) [15], which may open new directions in this area of research.

The aims of present work are to:

- Obtain the components of velocity, pressure drop, wall shear stress, temperature profile and mass concentration distribution.
- Study the heat transfer coefficient (Nusselt number) and the mass transfer coefficient (Sherwood number) at the porous wall of channel with uniform wall suction.

2. Mathematical formulation of the Problem

We consider the flow of a steady, non-isothermal, laminar, highly viscous incompressible Newtonian fluid in a narrow long permeable channel of height H and length L. A rectangular coordinate system (x,y,z) is chosen with the x-axis aligned with the midway and y- axis being taken perpendicular to the walls of the channel, Fig. (1). At $x=0$ fluid enters the channel having axial flow rate Q_0, then some of the fluid at $y=H$ is leaked across the walls of the channel with uniform permeation rate and remaining Q_L leaves at $x=L$. The walls of the channel is maintained at temperature T_w and the average temperature of the fluid is T_a. Moreover, the average concentration of the fluid is C_0, while C_w is wall concentration. The channel is assumed long ($H << L$) enough to neglect the end effects. The flow in this situation becomes two dimensional and the velocity profile is given by the following equation:

$$\mathbf{V} = [u(x,y), v(x,y)], \tag{2.1}$$

where, u and v are the velocities in the radial and axial directions, respectively.

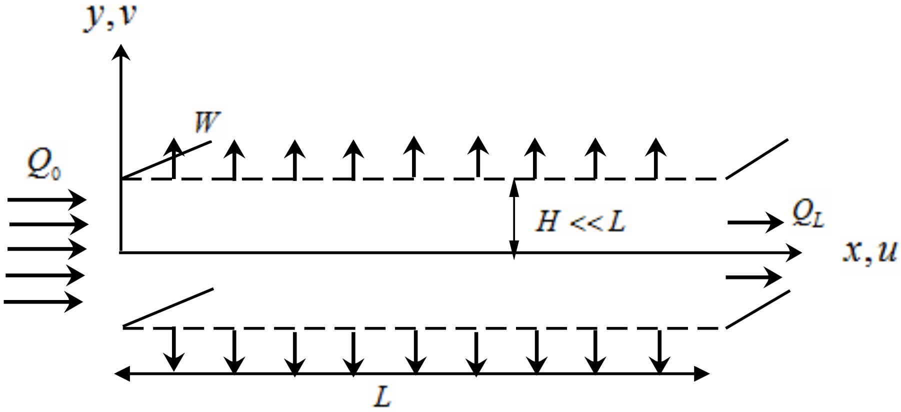

Fig. (1): Geometry of the problem.

The equations governing the motion of an incompressible and non-isothermal Newtonian fluid through a channel are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y} = 0. \tag{2.2}$$

$$\rho\left[u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right] = -\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right), \tag{2.3}$$

$$\rho\left[u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right] = -\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right), \tag{2.4}$$

$$\rho C_p\left[u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right] = k\left[\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right]+\mu\left[4\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2\right], \tag{2.5}$$

$$\left[u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}\right] = D\left[\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right]+\frac{DK_T}{T_m}\left[\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right], \tag{2.6}$$

where, p is the hydrodynamic pressure, ρ is the density, k is thermal conductivity, C_p is the specific heat at constant pressure, D is the coefficient of mass diffusivity, K_T is the thermal-diffusion ratio, and T_m is the mean temperature and μ is the viscosity of the fluid. The boundary conditions are taken as

$$v(x,H) = V_0, \tag{2.7}$$

$$u(x,H) = 0, \tag{2.8}$$

$$T(x,H) = T_w, \tag{2.9}$$

$$C(x,H) = C_w, \tag{2.10}$$

$$v(x,0) = 0, \tag{2.11}$$

$$\frac{\partial u(x,0)}{\partial y} = 0, \tag{2.12}$$

$$\frac{\partial T(x,0)}{\partial y} = 0, \tag{2.13}$$

$$\frac{\partial C(x,0)}{\partial y} = 0, \tag{2.14}$$

$$2W\int_0^H u(0,y)dy = Q_0, \tag{2.15}$$

$$p(0,y) = p_0, \tag{2.16}$$

where, V_0 is suction/injection velocity across the wall of the channel. Using the dimensionless quantities

$$\begin{aligned} x^* &= \frac{x}{L}, \quad y^* = \frac{y}{H}, \quad u^* = \frac{HLu}{Q_0}, \quad v^* = \frac{L^2 v}{Q_0}, \quad p^* = \frac{H^3 p}{\mu Q_0} \\ \theta &= \frac{T - T_w}{T_w - T_a}, \quad P_e = \frac{\rho C_p Q_0}{kH}, \quad B_r = \frac{\mu Q_0^2}{k T_w H} \quad \delta = \frac{H}{L}, \\ R_e &= \frac{Q_0}{\nu L}, \quad \phi = \frac{C - C_w}{C_w - C_0}, \quad S_c = \frac{Q_0}{DL}, \quad S_r = \frac{D K_T T_w L}{Q_0 T_m (C_w - C_0)}, \end{aligned} \tag{2.17}$$

where, P_e, B_r, R_e, S_c and S_r are Peclet, Brinkman, Reynolds, Schmidt and Soret numbers. Equations (2.2-2.5), after removing * becomes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \tag{2.18}$$

$$\delta R_e \left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \right] = -\frac{\partial p}{\partial x} + \left(\delta^2 \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right), \tag{2.19}$$

$$\delta^3 R_e \left[u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} \right] = -\frac{\partial p}{\partial y} + \delta^2 \left(\delta^2 \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right), \tag{2.20}$$

$$\delta P_e \left[u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} \right] = \left[\delta^2 \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right] + B_r \left[4\delta^2 \left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial u}{\partial y} + \delta^2 \frac{\partial v}{\partial x} \right)^2 \right], \tag{2.21}$$

$$\delta \left[u\frac{\partial \phi}{\partial x} + v\frac{\partial \phi}{\partial y} \right] = \frac{1}{S_c} \left[\delta^2 \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right] + S_r \left[\delta^2 \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right], \tag{2.22}$$

The boundary conditions in dimensionless form are

$$v(x,1) = U_0, \tag{2.23}$$
$$u(x,1) = 0, \tag{2.24}$$
$$\theta(x,1) = 1, \tag{2.25}$$
$$\phi(x,1) = 1, \tag{2.26}$$
$$v(x,0) = 0, \tag{2.27}$$
$$\frac{\partial u(x,0)}{\partial y} = 0, \tag{2.28}$$
$$\frac{\partial \theta(x,0)}{\partial y} = 0, \tag{2.29}$$
$$\frac{\partial \phi(x,0)}{\partial y} = 0, \tag{2.30}$$
$$\int_0^1 u(0,y)dy = \frac{1}{2\gamma}, \tag{2.31}$$
$$p(0,y) = 1, \tag{2.32}$$

where, $U_0 = \dfrac{L^2 V_0}{Q_0}$, $\gamma = \frac{W}{L}$ and $p_0 = \dfrac{H^3 p_0}{\mu Q_0}$ are suction (reabsorption), ratio of width to length and entrance pressure parameters.

3. Solution of the problem

For the solution of the problem, δ and R_e are assumed small [23-30], above equations can be written as

$$\frac{\partial p}{\partial x} = \frac{\partial^2 u}{\partial y^2}, \tag{3.1}$$

$$0 = -\frac{\partial p}{\partial y}, \tag{3.2}$$

$$0 = \frac{\partial^2 \theta}{\partial y^2} + B_r \left(\frac{\partial u}{\partial y}\right)^2, \tag{3.3}$$

$$0 = \frac{\partial^2 \phi}{\partial y^2} + S_r S_c \frac{\partial^2 \theta}{\partial y^2}, \tag{3.4}$$

From equation (3.2), it is observed that $p = p(x)$ only. Therefore, the solution of equation (3.1) using appropriate boundary conditions (2.24) and (2.28) is obtained as

$$u(x,y) = \frac{1}{2}\frac{dp}{dx}\left(y^2 - 1\right). \tag{3.5}$$

Substituting above equation (2.30) in equation (2.18), the expression for transverse velocity with the help of boundary conditions (2.27) reduces to

$$v(x,y) = \frac{1}{2}\frac{d^2 p}{dx^2}\left(y - \frac{y^3}{3}\right). \tag{3.6}$$

Equations (3.5-3.6) cannot describe the motion of the fluid particles inside the channel because of the unknown quantity $\frac{dp}{dx}$. To evaluate $\frac{dp}{dx}$, the boundary condition (2.23) is used, which transforms equation (3.6) into the the following differential equation in terms of pressure

$$\frac{d^2 p}{dx^2} = 3U_0. \tag{3.7}$$

Integrate equation (3.7) twice, we get

$$p(x) = \frac{3}{2}U_0 x^2 + C_1 x + C_2, \tag{3.8}$$

After using condition (2.32) above equation becomes

$$p(x) = \frac{3}{2}U_0 x^2 + C_1 x + 1, \tag{3.9}$$

Now, C_1 is the only constant to determine. Combining equations (3.3) and (2.30) in equation (2.25), we find that

$$C_1 = -\frac{3}{2\gamma}. \tag{3.10}$$

So, the expression for pressure becomes

$$p(x) = \frac{3}{2}U_0 x^2 - \frac{3}{2\gamma}x + 1, \tag{3.11}$$

therefore, the expressions for velocity components are

$$u(x,y) = \frac{1}{2}(3U_0 x - \frac{3}{2\gamma})\left(y^2 - 1\right), \tag{3.12}$$

$$v(x,y) = \frac{3}{2}U_0\left(y - \frac{y^3}{2}\right), \tag{3.13}$$

After using equation (3.12) in equation (3.3), the expression for temperature profile inside the permeable channel is obtained as

$$\theta(x,y) = 1 - \frac{3}{4}\left(y^4 - 1\right)\left(U_0 x - \frac{1}{2\gamma}\right)^2 B_r. \tag{3.14}$$

Using equation (3.14) in equation (3.4), the resultant equation is solved with conditions (2.26) and (2.30) and expression for concentration profile inside the permeable channel becomes

$$\phi(x,y) = 1 + \frac{3}{4}\left(y^4 - 1\right) S_r S_c B_r \left(U_0 x - \frac{1}{2\gamma}\right)^2. \tag{3.15}$$

3.1. *Flow rate*

The flow rate inside the channel can be calculated by using the formula

$$\bar{Q}(x) = \frac{Q(x)}{Q_0} = 2\gamma \int_0^1 u(x,y)dy, \tag{3.16}$$

where, $Q\bar{(x)}$ is dimensional flow rate. Using equation (3.12) in above equation (3.16), the flow rate becomes

$$Q(z) = 1 - 2U_0\gamma x. \tag{3.17}$$

which decreases along the channel.

3.2. *Average pressure*

The average pressure inside the channel can be calculated by using the formula

$$p\bar{(z)} = 2\int_0^1 p(x)dr = p(x), \tag{3.18}$$

The pressure drop inside the tube can now be calculated using the following formula

$$\Delta p = p\bar{(0)} - p\bar{(z)}, \tag{3.19}$$

using equation (3.4) in above formula, we get

$$\Delta p = \frac{3}{2\gamma}(1 - \gamma U_0), \tag{3.20}$$

which depends on U_0.

3.3. *Wall shear stress*

The wall shear stress is calculated by

$$\begin{aligned} \tau_w &= -\tau_{yx}\Big|_{r=1} \\ &= -\frac{\partial u}{\partial y}\Big|_{r=1} \end{aligned} \tag{3.21}$$

Using equation (3.5) in above relation we find

$$\tau_w = \frac{3}{2\gamma} - 3U_0 x. \tag{3.22}$$

Equation (3.22) shows that wall shear stress decays along the channel.

3.4. *Leakage flux*

The leakage flux is obtained as

$$q(x) = \frac{dQ(z)}{dx} = 2U_0\gamma, \tag{3.23}$$

that shows that the constant leakage across the walls inside the length of the channel.

3.5. *Fractional suction*

The amount of fluid filtered from the permeable walls is calculated by using the relation

$$FS = \frac{Q\bar{(0)} - Q\bar{(1)}}{Q\bar{(0)}} = 2U_0\gamma \tag{3.24}$$

For 80% fractional suction, the value of U_0 is obtained as $U_0 = 0.40\gamma$.

3.6. *Heat transfer*

Heat is transported mainly by conduction near the walls, so heat transfer coefficient is the amount of energy that flows through a unit area per unit time, such as

$$\begin{aligned} \ddot{q} &= -k\left(\frac{\partial T}{\partial y} + \frac{\partial T}{\partial x}\right)_{r=H} \\ \dot{q} &= -\left(\delta\frac{\partial \theta}{\partial x} + \frac{\partial \theta}{\partial y}\right)_{r=1} \end{aligned} \tag{3.25}$$

where $\ddot{q}$ is dimensional heat transfer rate also called Nusselt number, while $\dot{q} = \frac{\ddot{q}L}{T_0 k}$ is dimensionless heat transfer rate. Sine δ is small so, by using equation (3.7) in equation (3.16) we get

$$\dot{q} = \frac{3}{4}(2U_0x - 1)^2 B_r \tag{3.26}$$

3.7. *Mass transfer*

The mass transfer coefficient at the wall is defined as

$$\begin{aligned} J &= -D\left(\frac{\partial C}{\partial y} + \frac{\partial C}{\partial x}\right)_{r=H} \\ \dot{J} &== -\frac{DC_w J}{H}\left(\delta\frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial y}\right)_{r=1} \end{aligned} \tag{3.27}$$

where $\dot{J}$ is dimensional mass transfer rate also known as Sherwood number, while $\dot{J} = \frac{DC_w}{H}$ is dimensionless mass transfer rate. Sine δ is small so, by using equation (3.7) in equation (3.16) we get

$$\dot{J} = -\frac{3}{4}S_c S_r\,(2U_0x - 1)^2 B_r, \tag{3.28}$$

which depends on involved parameters.

4. Results and discussion

The current research has been carried out to study the heat and mass transfer of non isothermal viscous fluid in a permeable channel. The influence of suction parameter U_0 on the dimensionless longitudinal and transverse velocities $u(x,y)$ and $v(x,y)$, temperature $\theta(x,y)$, concentration $\phi(x,y)$, flow rate $Q(x)$, pressure drop $p(x)$, wall shear stress τ_w, heat transfer rate (Nusselt number) and mass transfer rate (Sherwood number) are illustrated in Figs. (2-10). The effects of Brinkman number on temperature $\theta(x,y)$, concentration $\phi(x,y)$, heat transfer rate (Nusselt number) and mass transfer rate (Sherwood number) are illustrated in Figs. (11-14)

The influence of U_0 on the dimensionless longitudinal and transverse velocities at different locations along the channel is illustrated in Fig. 2. It is perceived that the $u(x,y))$ is higher at entrance region as compared to the middle position and the exit region of the channel, which confirms the fluid leakage across the channel walls (Fig. 2). The velocity $u(x,y)$ reduces along the tube with an increase in U_0, while, opposite behavior is observed for the $v(r,z)$ at all the locations in side the channel (Fig. 3). The flow rate decreases downstream and it is higher at the exit for smaller value of U_0, which shows less leakage of the fluid through the channel walls, see Fig.4. The pressure drop decreases from entrance region to the exit region and at the exit region, $p(x)$ is higher for higher value of U_0 because the amount of fluid become less due to leakage see Fig.5. While opposite behaviour of wall shear stress in noted as compared to pressure drop and similar nature is observed as compared to flow rate, see Fig. 6 and Figs. (4-5).
The influence of U_0 on the dimensionless $\theta(x,y)$ at different locations are explained in Fig. 7 . The effects of U_0 on the dimensionless $\theta(x,y)$ are found to be highest at the center and then reduces to minimum at the surface. As U_0 increases, the dimensionless $\theta(x,y)$ decreases at each axial location. The variation of dimensionless $\phi(x,y)$ in transverse direction with U_0 can be visualized in Fig. 8 at three different locations. It is inferred that the dimensionless concentration increases from center to the surface and for higher values of the U_0, the dimensionless $\phi(r,z)$ increases from the center to the surface at both locations. The variation of heat transfer rate in axial direction with U_0 is illustrated in Fig. 9. It is noticed that heat transfer rate is higher at the inlet and it decreases along an axial direction. Figs. 10 display the variation of mass transfer rate in axial direction with U_0 at three different locations. It is noticed that mass transfer rate is found to be higher at the exit region and it increases with an increase in the U_0.
The effect of B_r on dimensionless $\theta(x,y)$ at different locations are explained in Fig. 11. It is observed that the strength of B_r increases the temperature profile at different locations inside the channel. Also temperature profile at the entrance region is higher than the middle and exit region of the channel. The dimensionless concentration $\phi(x,y)$ decreases with increasing B_r see Fig. 12 and its profile at the exit region is higher than middle and entrance region of the channel. With increasing B_r, heat transfer increases and its strength at the entrance region is higher than exit region, see Fig. 13, while opposite behaviour is observed for mass transfer inside the channel, see Fig. 14.

5. Concluding remarks

The steady flow of a non-isothermal, highly viscous laminar and incompressible Newtonian fluid is considered in narrow long a permeable channel.
From the theoretical point of view the following conclusions can be drawn.

1. Transverse velocity component is higher at the entrance region as compared to middle position and exit region of the channel.
2. With increasing the strength of suction parameter transverse velocity decreases while axial velocity increases at different locations inside the channel.
3. Flow rate, pressure drop and wall shear stress decreases from entrance region to exit region and with increasing suction parameter flow rate and wall shear stress decreases at the exit region.
4. 80% fractional suction is obtained by the relation $U_0 = 0.40\gamma$.
5. Temperature field decreases at the centerline of the channel with increasing suction parameter.
6. With increasing suction parameter concentration decreases from surface to the center of the channel and with increasing suction parameter concentration increases.
7. Heat transfer rate at the wall decreases from entrance to the exit region of the channel, also for smaller value of suction parameter, it is higher at the exit region.
8. Mass transfer rate at the wall increases inside the channel and it is higher for higher values of suction parameter at the exit region of the channel.
9. Temperature field increases and concentration profile decreases with increasing Brinkman number .

It should be mentioned that this work is theoretical in nature and significant in analyzing the heat and mass transfer of glomerular filtrate in proximal convoluted tubule, purification of blood in artificial kidney, fluid flow in porous micro channels and filtration of liquid using biological and industrial membranes.

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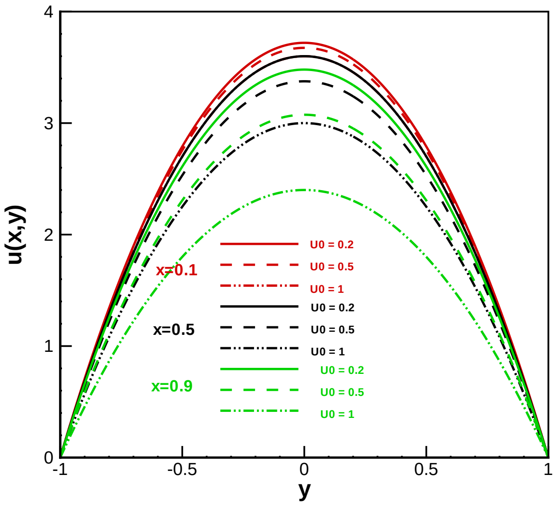

Fig. (2): Effect of U_0 on longitudinal velocity at different location $(x = 0.1, 0.5, 0.9)$, when $\gamma = 0.2$

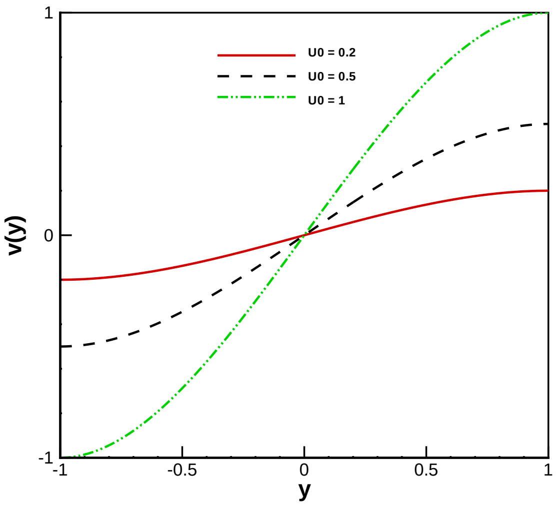

Fig. (3): Effect of U_0 on transverse velocity all locations inside the channel, when $\gamma = 0.2$

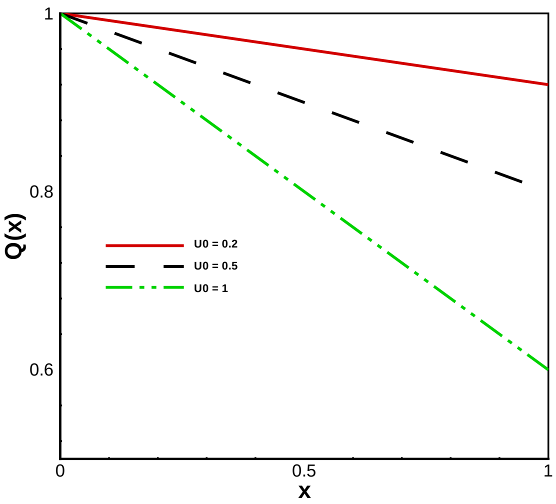

Fig. (4): Effect of U_0 on flow rate along the channel, when $\gamma = 0.2$

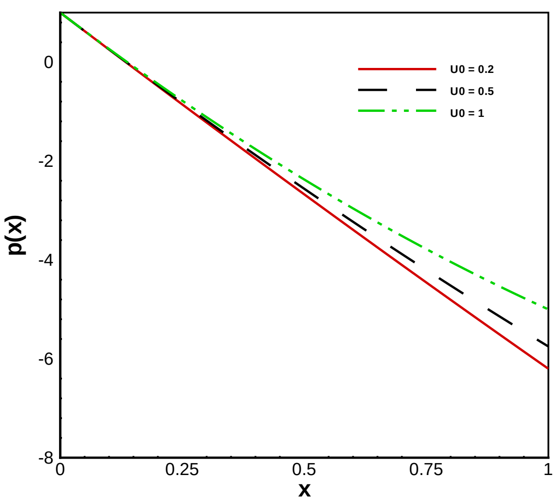

Fig. (5): Effect of U_0 on pressure along the channel, when $\gamma = 0.2$

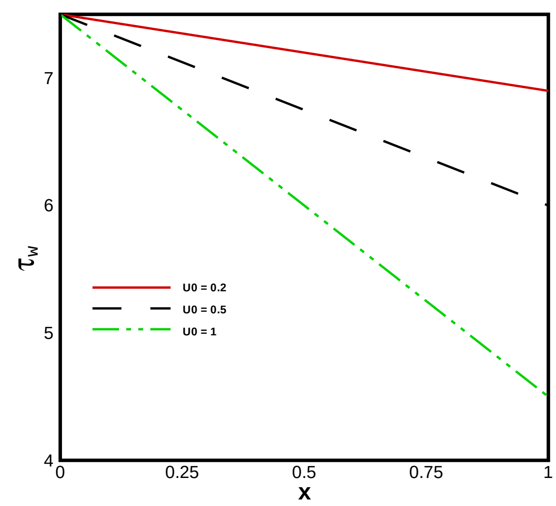

Fig. (6): Effect of U_0 on wall shear stress along the channel, when $\gamma = 0.2$

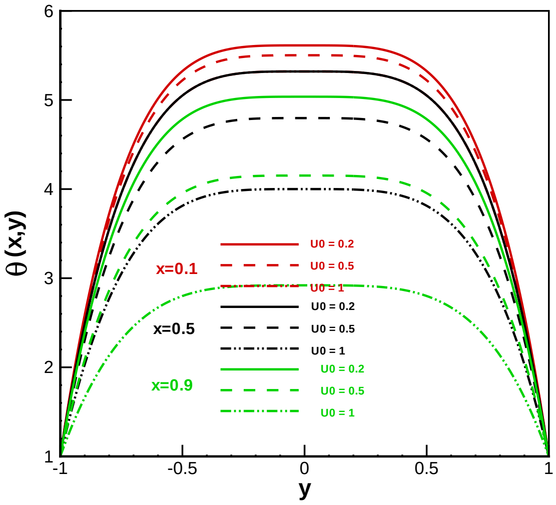

Fig. (7): Effect of U_0 on temperature at different location $(x = 0.1, 0.5, 0.9)$, when $\gamma = 0.2$, $B_r = 1$

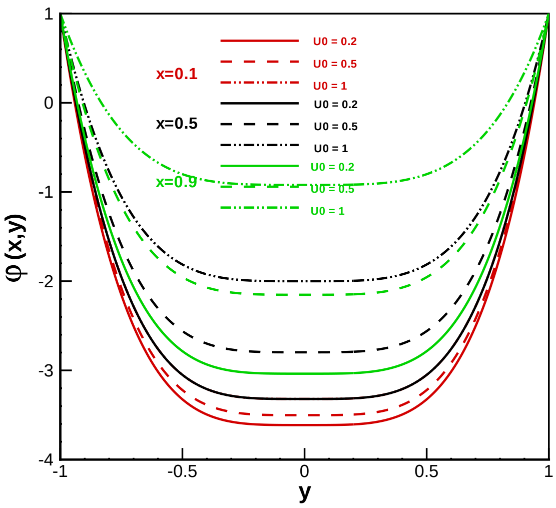

Fig. (8): Effect of U_0 on concentration at different location $(x = 0.1, 0.5, 0.9)$, when $\gamma = 0.2$, $B_r = 1$, $S_c = 1$, $S_r = 1$

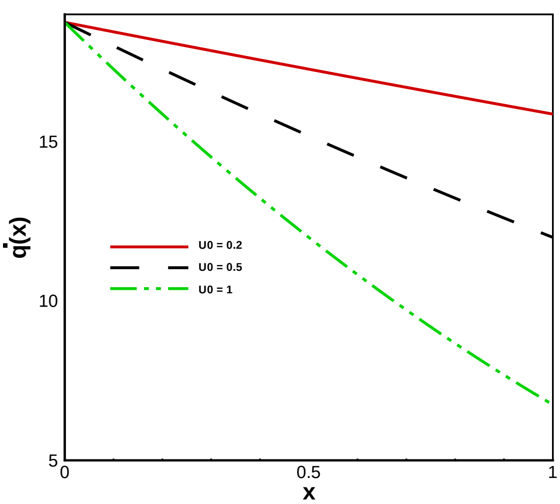

Fig. (9): Effect of U_0 on heat transfer rate inside the channel, when $\gamma = 0.2$, $B_r = 1$

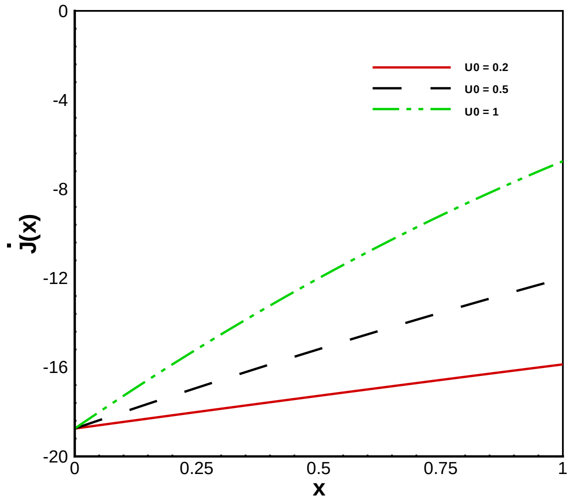

Fig. (10): Effect of U_0 on mass transfer rate inside the channel, when $\gamma = 0.2$, $B_r = 1$, $S_c = 1$, $S_r = 1$.

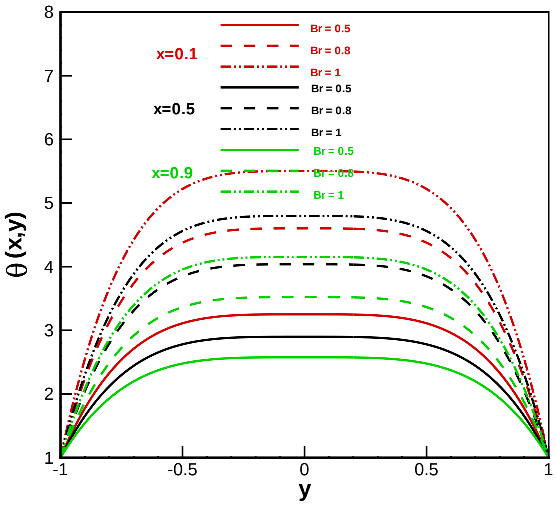

Fig. (11): Effect of B_r on temperature at different location $(x = 0.1, 0.5, 0.9)$, when $\gamma = 0.2$, $U_0 = 0.5$.

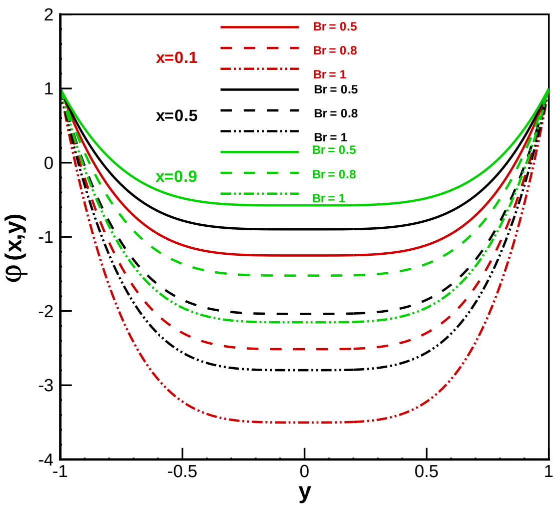

Fig. (12): Effect of B_r on concentration at different location $(x = 0.1, 0.5, 0.9)$, when $\gamma = 0.2$, $B_r = 0.5$.

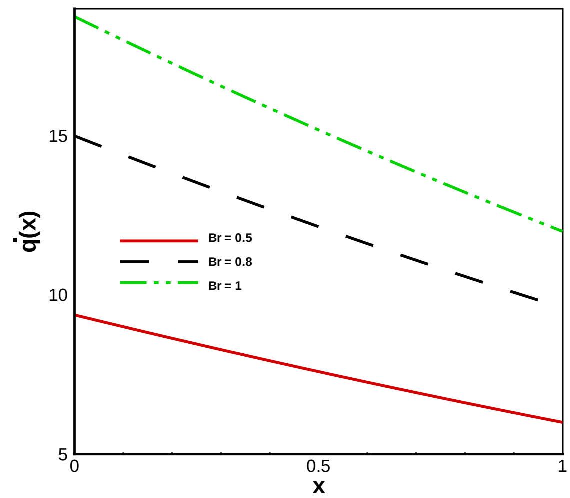

Fig. (13): Effect of B_r on heat transfer rate inside the channel, when $\gamma = 0.2$, $U0 = 0.5$

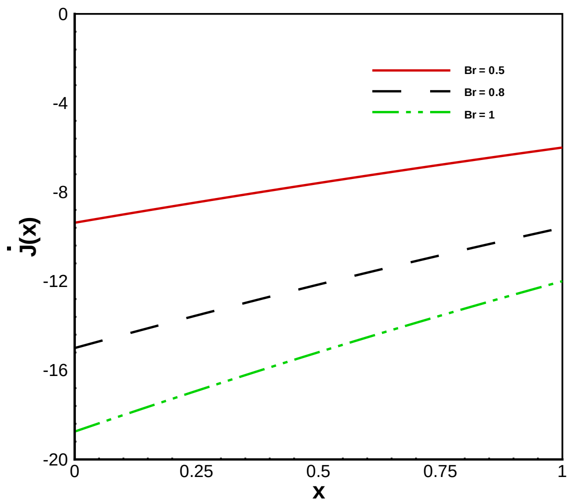

Fig. (14): Effect of B_r on mass transfer rate inside the channel, when $\gamma = 0.2$, $U0 = 0.5$