IJEMD-CSAI, 1 (2) (2022), 13 – 25 https://doi.org/10.54938/ijemdcsai.2022.01.2.138

International Journal of Emerging Multidisciplinaries:
Computer Science & Artificial Intelligence

Research Paper
Journal Homepage: www.ojs.ijemd.com
ISSN (print): 2791- 0164 ISSN (online): 2957-5036

An Application to Computer Science via New Fixed Point Technique

Khaleel Ahmad[1,*], Umar Ishtiaq[2], Jawad Afzal[3]

[1]*Department of Mathematics and statistics, International Islamic University Islamabad, Pakistan*
[2]*Office of Research, Innovation and Commercialization, University of Management and Technology, Lahore, Pakistan*
[3]*Centre for Advanced Studies in Pure and Applied Mathematics, Bahauddin Zakariya University Multan, Pakistan.*
**Corresponding author*

Abstract

We construct some standard orthogonal L-fuzzy fixed-point findings for almost orthogonal Θ-contraction in the context of orthogonal complete ms using the orthogonal L-fuzzy mapping (OLF-mapping). We prove some non-trivial examples to supporting our main results. Also, we establish a theoretical computer science application to domain of words.

Keywords: common fixed point; Θ-contraction; orthogonal metric space; domain of words.

1. Introduction and Preliminaries

Looking into fixed point (fp) theory's applications allows one to get a quick glimpse of the theory's breadth in several areas. According to certain conditions, fp theorems require that the functions have at least one fp. As can be seen, these findings are frequently advantageous to the field of mathematics and are essential to examining the existence and distinctiveness of solutions to several mathematical models.

In the context of metric space (ms) fp theory, Banach and Caccioppoli developed the fp theorem, which Banach [1] first proposed in 1922 and Caccioppoli [2] derived in 1931. Some scientists devised various conditions to discover fps. The fp theorem of Banach and Caccioppoli ensured that, in some cases, the function must have a fp if it was seized. Following this incredible finding by Banach and Caccioppoli, the fp theory has been effective in drawing in a sizable research community. To tackle problems related to real-world becomes evidently uncomplicated with the concept of fuzzy set (fs) theory developed in 1965 by Zadeh [3], as it is useful to the explanation of obscurity and inaccuracy. Arora, and Sharma [4] used fuzzy mappings to establish the common fps in the context of a metric linear space. In 1967, Goguen [5] replaced

Email addresses: khalil7066616@gmail.com (K. Ahmad), umarishtiaq000@gmail.com (U. Ishtiaq), jawad.afzal@bzu.edu.pk (J. Afzal)

the interval [0, 1] with the L-fs theory, modifying this idea. Fundamentally, there are two ways to interpret what L means:

- When L is a complete lattice equipped with a multiplication operator that satisfies several conditions, as established in [5].
- When L is a fully distributive complete lattice with an order-reversing involution.

Abdullahi and Azam [6] established the existence of common L-fuzzy fp theorems for L-fuzzy mappings (LF-mapping) fulfilling a rational expression by using the Hausdorff metric on L-fs. Adibi et al. [7] introduced several number of significant common fp theorems in L-fuzzy ms and introduced the idea of compatible mappings (cms) of type (P) in L-fuzzy ms, which is equivalent to the idea of compatible and cms of type (A) under certain circumstances, and derived some relationships between these mappings. They also established a coincidence point theorem and a fp theorem for cms of type (P). The terms, {α, ξ}-contractive, and α -admissible mappings on complete ms were introduced to state-related fp theorems by Ahmad et al. [8]. Additionally, several fp results for locally expansive and contractive mappings in complete ms were discovered. Iterations were used by Ahmad et al. [9, 10] to demonstrate the existence of common fuzzy fps for a series of locally contractive fuzzy mappings meeting generalized Banach-type contraction criteria.

By imposing a straightforward condition on the Θ function, Ahmad et al. [11] expand the finding of [M. Jleli, B. Samet, J. Inequal. Appl., 2014 (2014), 8 pages], and they also established some significant fp theorems for Suzuki-Berinde type Θ-contractions. [12] investigated JS contractions and developed several common fp theorems for them in the context of complete ms. In metric and ordered ms, Hussain et al. [22] developed the generalized α-GF-contraction and established Wardowski and Suzuki-type fp findings. Common fp theorems for fuzzy mappings under the assumption of contraction on a ms with the d_∞-metric (caused by the Hausdorff metric) on the family of fss were developed by Azam et al. [23].

The term "orthogonal set" (OS) was first used by Gordji et al. [13] who also expanded upon the Banach fp theorem. The concept of an orthogonal structure and the Banach contraction principle were developed by Uddin et al. [14, 15]. The orthogonal neutrosophic ms, which is an extension of the neutrosophic ms, was presented by Ishtiaq et al. [16] who also demonstrated some significant fp results in the context of the orthogonal neutrosophic ms. The idea of intuitionistic fuzzy double controlled ms was first suggested by Farheen et al. [17], who also expanded upon the idea of intuitionistic fuzzy b-ms.

With the help of the concept of orthogonal LF- mapping, we expand the findings of Ahmad et al. [18] in the context of orthogonal complete ms, and we develop some standard orthogonal L-fuzzy fp results for almost orthogonal Θ-contraction in this context. A theoretical computer science application is also given to demonstrate the importance of the findings.

Definition 1.1: [13] Suppose $\Xi \neq \emptyset$ be a set and $\perp$ be a binary relation on $\Xi \times \Xi$. If there exists an element $\varpi_1 \in \Xi$ such that below circumstance varifies:

$$(\text{ for all } \delta \in \Xi \ \varpi_1 \perp \delta) \text{or } (\text{ for all } \delta \in \Xi \ \delta \perp \varpi_1),$$

then the element ϖ_1 is said to be an orthogonal element and Ξ is an OS.

Definition 1.2: [13] Suppose $(\Xi, \perp)$ be an OS and (Ξ, σ) be a ms. Then $(\Xi, \perp, \sigma)$ be an orthogonal ms (oms).

Definition 1.3: [13] Suppose $(\Xi, \perp)$ be an OS. A sequence $\{\varpi_\rho\}$ is named an orthogonal sequence (O-Sequence) if

$$\left(\forall\, \rho \in \mathbb{N}, \varpi_\rho \perp \varpi_{\rho+1}\right) \text{ or } \left(\forall\, \rho \in \mathbb{N}, \varpi_{\rho+1} \perp \varpi_\rho\right).$$

Likewise, a Cauchy sequence $\{\varpi_\rho\}$ is called a Cauchy O-sequence if

$$\left(\forall\, \rho \in \mathbb{N}, \varpi_\rho \perp \varpi_{\rho+1}\right) \text{ or } \left(\forall\, \rho \in \mathbb{N}, \varpi_{\rho+1} \perp \varpi_\rho\right).$$

Definition 1.4: [13] Suppose $(\Xi, \perp)$ be an OS. A mapping $\xi_\perp : \Xi \to \Xi$ is said to be an orthogonal preserving (O-Preserving) if $\xi_\perp \varpi \perp \xi_\perp \delta$ whence $\varpi \perp \delta$.

Definition 1.5: [13] Suppose $(\Xi, \perp, \sigma)$ be an OMS. Then $\xi_\perp : \Xi \to \Xi$ is called an orthogonal continuous (O-continuous) at $\varpi \in \Xi$ if, for each O-sequence $\{\varpi_\rho\}$ in Ξ with $\{\varpi_\rho\} \to \varpi$, we have $\xi_\perp \varpi_\rho \to \xi_\perp \varpi$. Also, $\xi_\perp$ is said to be O-continuous on Ξ if, $\xi_\perp$ is O-continuous at each $\varpi \in \Xi$.

Definition 1.6: [13] Suppose $(\Xi, \perp, \sigma)$ be an OMS. Then Ξ is named an OCMS if every Cauchy O-sequence is convergent in Ξ.

Definition 1.7: [5] Suppose $(\Delta, \preccurlyeq)$ is a partially ordered set then $(\Delta, \preccurlyeq)$ is named as

i. a lattice if $\partial_1 \vee \partial_2 \in \Delta, \partial_1 \wedge \partial_2 \in \Delta$ for each $\partial_1, \partial_2 \in \Delta$.
ii. A complete lattice, if $\vee A \in \Delta, \wedge A \in \Delta$ for any $A \subseteq \Delta$.
iii. Distributive lattice if $\partial_1 \vee (\partial_2 \wedge \partial_3) = (\partial_1 \vee \partial_2) \wedge (\partial_1 \vee \partial_3)$ and $\partial_1 \wedge (\partial_2 \vee \partial_3) = (\partial_1 \wedge \partial_2) \vee (\partial_1 \wedge \partial_3)$ for any $\partial_1, \partial_2, \partial_3 \in \Delta$.

Definition 1.8: [5] Suppose a lattice with 1_Δ and 0_Δ and let $\partial_1, \partial_2 \in \Delta$. So ∂_2 is named as a complement of ∂_1, if $\partial_1 \vee \partial_2 = 1_\Delta, \partial_1 \wedge \partial_2 = 0_\Delta$. If $\partial \in \Delta$ then ∂' is unique complement element.

Definition 1.9: [5] A L-fs A on a nonempty set Ξ is a function $A: \Xi \to \Delta$, where Δ is satisfied (ii) and (iii) in the definition 1.7 with 1_Δ and 0_Δ.
ϖ_{F_Δ} admissible for a pair of LF-mappings is a notion that was introduced by Azam et al. [19] in 2014 and was then used to develop a common L-fuzzy fp theorem.
Definition 1.10: [19] Let Ξ_1 be an arbitrary set, Ξ_2 be a ms. If Q is a mapping from Ξ_1 into $\Im_\Delta(\Xi_2)$ then Q is LF-mapping. An LF-mapping Q is a Δ-fuzzy subset on $\Xi_1 \times \Xi_2$ with membership function $Q(\varpi)(v)$. So, $Q(\varpi)(v)$ is the grade of membership of v in $Q(\varpi)$.

Definition 1.11: [19] Suppose (Ξ, σ) be a ms and $\mathfrak{D}, Q: \Xi \to \Im_\Delta(\Xi)$ are LF-mappings. If $\varpi^* \in [Q\varpi^*]_{\alpha_\Delta}$ then an element $z \in \Xi$ is L-fuzzy fp of Q. where $\alpha_\Delta \in \Delta\{0_\Delta\}$. The point $\varpi^* \in \Xi$ is called a common Δ-fuzzy fp of $\mathfrak{D}$ and Q if $\varpi^* \in [\mathfrak{D}\varpi^*]_{\alpha_\Delta} \cap [Q\varpi^*]_{\alpha_\Delta}$. When $\alpha_\Delta = 1_\Delta$, it is called a common fp of LF-mappings.

In 2015, Jleli et al. [20] gave the notion of Θ-contractions and proved some new fp results for such contractions in the setting of generalized ms.

Definition 1.12:[20] Let $\Theta: (0,\infty) \to (1,\infty)$ be a function satisfying:

(Θ_1) Θ is non-decreasing.

(Θ_2) For each sequence $\{\alpha_\rho\} \subseteq R^+$, $\lim_{\rho\to\infty} \Theta(\alpha_\rho) = 1$ if and only if $\lim_{\rho\to\infty}(\alpha_\rho) = 0$;

(Θ_3) There exists 0 < r < 1 and $l \in (0, \infty]$ such that

$$\lim_{\alpha\to 0^+}\left(\frac{\Theta(\alpha)-1}{\alpha^r}\right) = l.$$

A mapping $\mathfrak{D} : \Xi \to \Xi$ is said to be Θ -contraction if there exist the function Θ satisfying $(\Theta_1) - (\Theta_3)$ and a constant $k \in (0,1)$ such that for all $\varpi;\ \nu \in \Xi$

$$\sigma(\mathfrak{D}\varpi, \mathfrak{D}\nu) > 0 \Rightarrow \Theta(\sigma(\mathfrak{D}\varpi, \mathfrak{D}\nu)) \leq [\Theta(\sigma(\varpi, \nu))]^k$$

Definition 1.13: Let (Ξ, σ) be a ms and $\mathrm{CB}(\Xi)$ be the family of nonempty, closed and bounded subsets (BS) of Ξ. For $A, B \in \mathrm{CB}(\Xi)$, define

$$\Pi(A;B) = \max\left\{\sup_{\partial\in A} \sigma(\partial, B), \sup_{b\in B}(b, A)\right\}$$

where

$$\sigma(\partial, A) = \inf_{\nu\in A} \sigma(\varpi, \nu).$$

Lemma 1.1: [19] Let (Ξ, σ) be a ms and $A, B \in \mathrm{CB}(\Xi)$, then for each $\partial \in A$,

$$\sigma(\partial, B) \leq \Pi(A;B).$$

2. Main Results:

In this section, we derive common orthogonal Δ-fuzzy fp theorems for almost orthogonal Θ -contraction in the setting of OCMS.

Definition 2.1: Suppose $(\Delta, \preccurlyeq)$ is a partially ordered set then $(\Delta, \preccurlyeq)$ is named as:

i. an orthogonal lattice if $\partial_1 \vee \partial_2 \in \Delta, \partial_1 \wedge \partial_2 \in \Delta$ for each $\partial_1 \perp \partial_2$ and $\partial_1, \partial_2 \in \Delta$.
ii. An orthogonal complete lattice, if $\vee A \in \Delta,\ \wedge A \in \Delta$ for any $A \subseteq \Delta$.
iii. An orthogonal distributive lattice if $\partial_1 \vee (\partial_2 \wedge \partial_3) = (\partial_1 \vee \partial_2) \wedge (\partial_1 \vee \partial_3)$ and $\partial_1 \wedge (\partial_2 \vee \partial_3) = (\partial_1 \wedge \partial_2) \vee (\partial_1 \wedge \partial_3)$ such that $\partial_1 \perp \partial_2, \partial_2 \perp \partial_3$ and $\partial_1 \perp \partial_3$ for any $\partial_1, \partial_2, \partial_3 \in \Delta$.

Definition 2.2: An orthogonal L-fs A on a O-set $\Xi \neq \phi$ is a function $A: \Xi \to \Delta$, where Δ is An orthogonal complete distributive lattice with 1_Δ and 0_Δ.

Definition 2.3: Let Ξ_1 be an arbitrary O-set, Ξ_2 be an OMS. If $Q: \Xi_1 \to \mathfrak{I}_\Delta(\Xi_2)$ is an orthogonal L-fuzzy mapping. An OLF-mapping Q is an orthogonal Δ -fuzzy subset on $\Xi_1 \times \Xi_2$ with membership function $Q(\varpi)(\nu)$. The function $Q(\varpi)(\nu)$ is the grade of membership of ν in $Q(\varpi)$.

Definition 2.4: Let $(\Xi, \perp, \sigma)$ be an OMS and $\mathfrak{D}, Q: \Xi \to \mathfrak{I}_\Delta(\Xi)$ be an OLF-mappings. An element $z \in \Xi$ is named as an orthogonal Δ-fuzzy fp of Q if $\varpi^* \in [Q\varpi^*]_{\alpha_\Delta}$ where $\alpha_\Delta \in \Delta\backslash\{0_\Delta\}$. The point $\varpi^* \in \Xi$ is named as a common orthogonal Δ-fuzzy fp of $\mathfrak{D}$ and Q if $\varpi^* \in [\mathfrak{D}\varpi^*]_{\alpha_\Delta} \cap [Q\varpi^*]_{\alpha_\Delta}$. When $\alpha_\Delta = 1_\Delta$, it is called a common fp of OLF-mappings.

Theorem 2.1: Let $(\Xi, \perp, \sigma)$ be an OCMS and $\mathfrak{O}$, $Q: \Xi \to \mathfrak{I}_{\Delta}(\Xi)$ be a pair of OLF-mappings, orthogonal L-fuzzy preserving, orthogonal continuous and for each $\alpha_{\Delta} \in \Delta \backslash \{0_{\Delta}\}$, $[\mathfrak{O}\varpi]_{\alpha_{\Delta}(\varpi)}, [Q\upsilon]_{\alpha_{\Delta}(\upsilon)}$ are nonempty close and BS of Ξ. If there exist some $\Theta \in F$, $k \in (0,1)$ and $\Delta \geq 0$ such that

$$\Pi\left([\mathfrak{O}\varpi]_{\alpha_{\Delta}(\varpi)}, [Q\upsilon]_{\alpha_{\Delta}(\upsilon)}\right) > 0,$$

$$\Rightarrow \Theta\left(\Pi\left([\mathfrak{O}\varpi]_{\alpha_{\Delta}(\varpi)}, [Q\upsilon]_{\alpha_{\Delta}(\upsilon)}\right)\right) \leq \Theta\big(\sigma(\varpi, \upsilon)\big)^{k} + \Delta m(\varpi, \upsilon) \quad (2.1)$$

for all $\varpi, \upsilon \in \Xi$ with $\varpi \perp \upsilon$, where

$$m(\varpi, \upsilon) = \min\left\{\begin{matrix}\sigma\left(\varpi, [\mathfrak{O}\varpi]_{\alpha_{\Delta}(\varpi)}\right), \sigma\left(\upsilon, [Q\upsilon]_{\alpha_{\Delta}(\upsilon)}\right), \\ \sigma\left(\varpi, [Q\upsilon]_{\alpha_{\Delta}(\upsilon)}\right), \sigma\left(\upsilon, [\mathfrak{O}\varpi]_{\alpha_{\Delta}(\varpi)} \right)\end{matrix}\right\}. \qquad (2.2)$$

Then $\mathfrak{O}$ and Q have a common L-fuzzy fp.

Proof: Let ϖ_0 be an arbitrary element in Ξ, (for all $\delta \in \Xi$ $\varpi_0 \perp \delta$) or (for all $\delta \in \Xi$ $\delta \perp \varpi_0$) then by hypotheses there exists $\alpha_{\Delta}(\varpi_0) \in \Delta\{0_{\Delta}\}$ such that $\varpi_0 \perp [\mathfrak{O}\varpi_0]_{\alpha_{\Delta}(\varpi_0)}$ or $[\mathfrak{O}\varpi_0]_{\alpha_{\Delta}(\varpi_0)} \perp \varpi_0$ and $[\mathfrak{O}\varpi_0]_{\alpha_{\Delta}(\varpi_0)}$ is a non empty closed bounded subset of Ξ and let $\varpi_1 \in [\mathfrak{O}\varpi_0]_{\alpha_{\Delta}(\varpi_0)}$. For this ϖ_1 there exists $\alpha_{\Delta}(\varpi_1) \in \Delta \backslash \{0_{\Delta}\}$ such that $[Q\varpi_1]_{\alpha_{\Delta}(\varpi_1)}$ is a nonempty, closed and bounded subset of Ξ. Since $\mathfrak{O}$ and Q are the orthogonal preserving then $\left([\mathfrak{O}\varpi_0]_{\alpha_{\Delta}(\varpi_0)} \perp [Q\varpi_1]_{\alpha_{\Delta}(\varpi_1)}\right)$ or $\left([Q\varpi_1]_{\alpha_{\Delta}(\varpi_1)} \perp [\mathfrak{O}\varpi_0]_{\alpha_{\Delta}(\varpi_0)}\right)$ by using Lemma 1.1, (Θ_1) and equation (2.1), we get

$$\Theta\left(\sigma\left(\varpi_1, [Q\varpi_1]_{\alpha_{\Delta}(\varpi_1)}\right)\right) \leq \Theta\left(\Pi\left([\mathfrak{O}\varpi_0]_{\alpha_{\Delta}(\varpi_0)}, [Q\varpi_1]_{\alpha_{\Delta}(\varpi_1)}\right)\right) \leq \Theta\big(\sigma(\varpi_0, \varpi_1)\big)^{k} + \Delta m(\varpi_0, \varpi_1), \quad (2.3)$$

where

$$m(\varpi_0, \varpi_1) = \min\left\{\begin{matrix}\sigma\left(\varpi_0, [\mathfrak{O}\varpi_0]_{\alpha_{\Delta}(\varpi_0)}\right), \sigma\left(\varpi_1, [Q\varpi_1]_{\alpha_{\Delta}(\varpi_1)}\right), \\ \sigma\left(\varpi_0, [Q\varpi_1]_{\alpha_{\Delta}(\varpi_1)}\right), \sigma\left(\varpi_1, [\mathfrak{O}\varpi_0]_{\alpha_{\Delta}(\varpi_0)}\right)\end{matrix}\right\}.$$

From (Θ_4), we know that

$$\Theta\left(\sigma\left(\left(\varpi_1, [Q\varpi_1]_{\alpha_{\Delta}(\varpi_1)}\right)\right)\right) = \inf_{\upsilon \in [Q\varpi_1]_{\alpha_{\Delta}(\varpi_1)}} \Theta\big(\sigma(\varpi_1, \upsilon)\big).$$

Thus from (2.3), we get

$$\inf_{\upsilon \in [Q\varpi_1]_{\alpha_{\Delta}(\varpi_1)}} \Theta\big(\sigma(\varpi_1, \upsilon)\big) \leq \Theta\big(\sigma(\varpi_0, \varpi_1)\big)^{k} + \min\left\{\begin{matrix}\sigma\left(u_0, [\mathfrak{O}\varpi_0]_{\alpha_{\Delta}(\varpi_0)}\right), \sigma\left(u_1, [Q\varpi_1]_{\alpha_{\Delta}(\varpi_1)}\right) \\ \sigma\left(u_0, [Q\varpi_1]_{\alpha_{\Delta}(\varpi_1)}\right), \sigma\left(u_1, [\mathfrak{O}\varpi_0]_{\alpha_{\Delta}(\varpi_0)}\right)\end{matrix}\right\}. \quad (2.4)$$

Then, from (2.4), there exist $\varpi_2 \in [Q\varpi_1]_{\alpha_{\Delta}(\varpi_1)}$ such that

$$\Theta(\sigma(\varpi_1, \varpi_2)) \leq \big[\Theta\big(\sigma(\varpi_0, \varpi_1)\big)\big]^{k} + \min\left\{\begin{matrix}\sigma(\varpi_0, \varpi_1), \sigma(\varpi_1, \varpi_2), \\ \sigma(\varpi_0, \varpi_2), \sigma(\varpi_1, \varpi_1)\end{matrix}\right\}.$$

Thus we have

$$\Theta(\sigma(\varpi_1, \varpi_2)) \leq \big[\Theta\big(\sigma(\varpi_0, \varpi_1)\big)\big]^{k}.$$

For this ϖ_2 there exists $\alpha_{\Delta}(\varpi_2) \in \Delta \backslash \{0_{\Delta}\}$ such that $[\mathfrak{O}\varpi_2]_{\alpha_{\Delta}(\varpi_2)}$ is a non-empty close and BS of Ξ. Using the Lemma 1.1, (Θ_1) and (2.1), we have

$$\Theta\left(\sigma\left(\varpi_2, [\mathfrak{O}\varpi_2]_{\alpha_{\Delta}(\varpi_2)}\right)\right) \leq \Theta\left(\Pi\left([Q\varpi_1]_{\alpha_{\Delta}(\varpi_1)}, [\mathfrak{O}\varpi_2]_{\alpha_{\Delta}(\varpi_2)}\right)\right)$$

$$= \Theta\left(\Pi\left([\mathfrak{O}\varpi_2]_{\alpha_{\Delta}(\varpi_2)}, [Q\varpi_1]_{\alpha_{\Delta}(\varpi_1)}\right)\right) \leq \big[\Theta\big(\sigma(\varpi_2, \varpi_1)\big)\big]^{k} + \Delta m(\varpi_2 + \varpi_1).$$

Thus, we get

$$\Theta\left(\sigma\left(\varpi_2, [\mathfrak{O}\varpi_2]_{\alpha_{\Delta}(\varpi_2)}\right)\right) \leq \big[\Theta\big(\sigma(\varpi_2, \varpi_1)\big)\big]^{k} + \Delta m(\varpi_2 + \varpi_1).$$

Where

$$m(\varpi_2, \varpi_1) = \min\left\{\begin{matrix}\sigma\left(\varpi_2, [\mathfrak{O}\varpi_2]_{\alpha_{\Delta}(\varpi_2)}\right), \sigma\left(\varpi_1, [Q\varpi_1]_{\alpha_{\Delta}(\varpi_1)}\right), \\ \sigma\left(\varpi_2, [Q\varpi_1]_{\alpha_{\Delta}(\varpi_1)}\right), \sigma\left(\varpi_1, [\mathfrak{O}\varpi_2]_{\alpha_{\Delta}(\varpi_2)}\right)\end{matrix}\right\}.$$

This further implies that

$$\Theta\left(\sigma\left(\varpi_2,[\mathfrak{D}\varpi_2]_{\alpha_\Delta(\varpi_2)}\right)\right)\leq\ \Theta[\sigma(\varpi_1,\varpi_2)]^k+\ \min\left\{\begin{matrix}\sigma\left(\mathrm{u}_2,[\mathfrak{D}\varpi_2]_{\alpha_\Delta(\varpi_2)}\right),\sigma\left(\mathrm{u}_1,[\mathcal{Q}\varpi_1]_{\alpha_\Delta(\varpi_1)}\right),\\ \sigma\left(\mathrm{u}_2,[\mathcal{Q}\varpi_1]_{\alpha_\Delta(\varpi_1)}\right),\sigma\left(\mathrm{u}_1,[\mathfrak{D}\varpi_2]_{\alpha_\Delta(\varpi_2)}\right)\end{matrix}\right\}.\quad (2.7)$$

From (Θ_4), we know that

$$\Theta\left(\sigma\left(\varpi_2,[\mathfrak{D}\varpi_2]_{\alpha_\Delta(\varpi_2)}\right)\right)=\inf_{\upsilon_1\in[\mathfrak{D}\varpi_2]_{\alpha_\Delta(\varpi_2)}}\Theta\left(\sigma(\varpi_2,\upsilon_1)\right).$$

$$\inf_{\upsilon_1\in[\mathfrak{D}\varpi_2]_{\alpha_\Delta(\varpi_2)}}\Theta\left(\sigma(\varpi_2,\upsilon_1)\right)\leq\Theta[\ \sigma(\varpi_1,\varpi_2)]^k$$
$$+\min\left\{\begin{matrix}\sigma\left(\varpi_2;[\mathfrak{D}\varpi_2]_{\alpha_\Delta(\varpi_2)}\ \right),\sigma\left(\varpi_1;[\mathcal{Q}\varpi_1]_{\alpha_\Delta(\varpi_1)}\ \right),\\ \sigma\left(\varpi_2;[\mathcal{Q}\varpi_1]_{\alpha_\Delta(\varpi_1)}\ \right),\sigma\left(\varpi_1;[\mathfrak{D}\varpi_2]_{\alpha_\Delta(\varpi_2)}\ \right)\end{matrix}\right\}.\quad (2.8)$$

Then, from (2.8), their exists $\varpi_3\in[\mathfrak{D}\varpi_2]_{\alpha_\Delta(\varpi_2)}$ such that

$$\Theta\left(\sigma(\varpi_2,\upsilon_3)\right)\leq\ \Theta[\ \sigma(\varpi_1,\varpi_2)]^k+\min\left\{\begin{matrix}\sigma(\varpi_2,\varpi_3\ ),\sigma(\varpi_1,\varpi_2\ ),\\ \sigma(\varpi_2,\varpi_2\ ),\sigma(\varpi_1,\varpi_3\ )\end{matrix}\right\}.$$

Then, we have

$$\Theta\left(\sigma(\varpi_2,\varpi_3)\right)\leq\left[\Theta\left(\sigma(\varpi_1,\varpi_2)\right)\right]^k.\quad (2.9)$$

So, continuing this process, we get $\{\varpi_\rho\}$ in Ξ such that $\varpi_{2\rho+1}\in\left[\mathfrak{D}\varpi_{2\rho}\right]_{\alpha_\Delta(\varpi_{2\rho})}$ and $\varpi_{2\rho+2}\in\left[\mathcal{Q}\varpi_{2\rho+1}\right]_{\alpha_\Delta(\varpi_{2\rho+1})}$ and

$$\Theta\left(\sigma\left(\varpi_{2\rho+1},\varpi_{2\rho+2}\right)\right)\leq\ \left[\Theta\left(\sigma\left(\varpi_{2\rho},\varpi_{2\rho+1}\right)\right)\right]^k\quad (2.10)$$

and

$$\Theta\left(\sigma\left(\varpi_{2\rho+2},\varpi_{2\rho+3}\right)\right)\leq\ \left[\Theta\left(\sigma\left(\varpi_{2\rho+1},\varpi_{2\rho+2}\right)\right)\right]^k\quad (2.11)$$

$\forall\rho\in\mathbb{N}$. from (2.10) and (2.11), we get

$$\Theta\left(\sigma\left(\varpi_\rho,\varpi_{\rho+1}\right)\right)\leq\ \left[\Theta\left(\sigma\left(\varpi_{\rho-1},\varpi_\rho\right)\right)\right]^k.\quad (2.12)$$

This further implies that

$$\Theta\left(\sigma\left(\varpi_\rho,\varpi_{\rho+1}\right)\right)\leq\ \left[\Theta\left(\sigma\left(\varpi_{\rho-1},\varpi_\rho\right)\right)\right]^k\leq\left[\Theta\left(\sigma\left(\varpi_{\rho-2},\varpi_{\rho-1}\right)\right)\right]^{k^2}\leq\cdots\leq\left[\Theta\left(\sigma(\varpi_0,\varpi_1)\right)\right]^{k^\rho}\quad (2.13)$$

$\forall\rho\in\mathbb{N}$. Since $\Theta\in F$, as $\rho\to\infty$ in (2.13), we get

$$\lim_{\rho\to\infty}\Theta\left(\sigma\left(\varpi_\rho,\varpi_{\rho+1}\right)\right)=1.\quad (2.14)$$

This implies that

$$\lim_{\rho\to\infty}\left(\sigma(\varpi_\rho,\varpi_{\rho+1})\right)=0. \quad (2.15)$$

Using the Definition 1.12, their exist $0<r<1$ and $l\in(0,\infty]$, such that

$$\lim_{\rho\to\infty}\frac{\Theta\left(\sigma(\varpi_\rho,\varpi_{\rho+1})\right)-1}{\sigma(\varpi_\rho,\varpi_{\rho+1})^r}=l. \qquad (2.16)$$

Assume $l<\infty$. In this situation, Suppose $\varpi=\frac{l}{2}>0$. Using limit, their exist $\rho_0\in\mathbb{N}$ such that

$$\left|\frac{\Theta\left(\sigma(\varpi_\rho,\varpi_{\rho+1})\right)-1}{\sigma(\varpi_\rho,\varpi_{\rho+1})^r}-l\right|\le\varpi,$$

$\forall\rho>\rho_0$. This implies that

$$\frac{\Theta\left(\sigma(\varpi_\rho,\varpi_{\rho+1})\right)-1}{\sigma(\varpi_\rho,\varpi_{\rho+1})^r}\ge l-\varpi=\frac{l}{2}=\varpi,$$

for all $\rho>\rho_0$. Then

$$\rho\sigma(\varpi_\rho,\varpi_{\rho+1})^r\le\alpha\rho\left[\Theta\left(\sigma(\varpi_\rho,\varpi_{\rho+1})\right)-1\right] \quad (2.17)$$

$\forall\rho>\rho_0$, where $=\frac{1}{\varpi}$. Then, we assume that when $l=\infty$. Let $\varpi>0$. Using the limit, there exists $\rho_0\in\mathbb{N}$ such that

$$\varpi\le\frac{\Theta\left(\sigma(\varpi_\rho,\varpi_{\rho+1})\right)-1}{\sigma(\varpi_\rho,\varpi_{\rho+1})^r},$$

$\forall\rho>\rho_0$. This implies that

$$\rho\sigma(\varpi_\rho,\varpi_{\rho+1})^r\le\alpha\rho\left[\Theta\left(\sigma(\varpi_\rho,\varpi_{\rho+1})\right)-1\right],$$

$\forall\rho>\rho_0$, where $=\frac{1}{\varpi}$. Then, in all cases, $\exists\,\alpha>0$ and $\rho_0\in\mathbb{N}$, we get

$$\rho\sigma(\varpi_\rho,\varpi_{\rho+1})^r\le\alpha\rho\left[\Theta\left(\sigma(\varpi_\rho,\varpi_{\rho+1})\right)-1\right], \quad (2.18)$$

$\forall\rho>\rho_0$. Thus by (2.13) and (2.18), we get

$$\rho\sigma(\varpi_\rho,\varpi_{\rho+1})^r\le\alpha\rho\left(\left[\Theta(\sigma(\varpi_0,\varpi_1))\right]^{r^\rho}-1\right). \quad (2.19)$$

Using limit as $\rho \to \infty$ in (2.19), we get

$$\lim_{\rho\to\infty} \rho\alpha\left(\varpi_\rho, \varpi_{\rho+1}\right)^r = 0.$$

Thus, $\exists\, \rho_1 \in \mathbb{N}$, such that

$$\alpha\left(\varpi_\rho, \varpi_{\rho+1}\right) \leq \frac{1}{\rho^{\frac{1}{r}}} \qquad (2.20)$$

$\forall\, \rho > \rho_1$. we show that $\{\varpi_\rho\}$ is a Cauchy sequence. For $m > \rho > \rho_1$, we get

$$\alpha\left(\varpi_\rho, \varpi_m\right) \leq \sum_{i=\rho}^{m-1} \alpha(\varpi_i, \varpi_{i+1}) \leq \sum_{i=\rho}^{m-1} \frac{1}{i^{\frac{1}{r}}} \leq \sum_{i=\rho}^{\infty} \frac{1}{i^{\frac{1}{r}}}. \qquad (2.21)$$

Then, $0 < r < 1$, as $\sum_{i=1}^{\infty} \frac{1}{i^{\frac{1}{r}}}$ is converges. However, $\sigma(\varpi_\rho, \varpi_m) \to 0$ as $m, \rho \to \infty$. Thus, we proved that $\{\varpi_\rho\}$ is a Cauchy orthogonal sequence. An orthogonal completeness of $(\Xi, \perp, \sigma)$ ensures that $\exists\, \varpi^* \in \Xi$ such that $\left(\varpi_\rho \perp \varpi^*\right)$or $(\varpi^* \perp \varpi_\rho)$ and $\lim_{\rho\to\infty} \varpi_\rho \to \varpi^*$. Now, we prove that $\varpi^* \in [\mathcal{Q}\varpi^*]_{\alpha_\Delta(\varpi^*)}$. We suppose on the contrary that $\varpi^* \notin [\mathcal{Q}\varpi^*]_{\alpha_\Delta(\varpi^*)}$ then, there exist an element $\rho_0 \in \mathbb{N}$ and an orthogonal subsequence $\{\varpi_{\rho_k}\}$ of $\{\varpi_\rho\}$ such that $\sigma\left(\varpi_{2\rho_k+1}, [\mathcal{Q}\varpi^*]_{\alpha_\Delta(\varpi^*)}\right) > 0$ for all $\rho_k \geq \rho_0$. Since $\sigma\left(\varpi_{2\rho_k+1}, [\mathcal{Q}\varpi^*]_{\alpha_\Delta(\varpi^*)}\right) > 0$ for all $\rho_k \geq \rho_0$, so by (Θ_1), we have

$$1 < \Theta\left[\sigma\left(\varpi_{2\rho_k+1}, [\mathcal{Q}\varpi^*]_{\alpha_\Delta(\varpi^*)}\right)\right] \leq \Theta\left[\Pi\left(\left[\mathfrak{D}\varpi_{2\rho_k}\right]_{\alpha_L(\varpi_{2\rho_k})}, [\mathcal{Q}\varpi^*]_{\alpha_\Delta(\varpi^*)}\right)\right]$$

$$\leq \left[\Theta\left(\sigma\left(\varpi_{2\rho_k}, \varpi^*\right)\right)\right]^k + \min\left\{\begin{matrix}\sigma\left(\varpi_{2\rho_k}; \left[\mathfrak{D}\varpi_{2\rho_k}\right]_{\alpha_\Delta\left(\varpi_{2\rho_k}\right)}\right), \sigma\left(\varpi^*; [\mathcal{Q}\varpi^*]_{\alpha_\Delta(\varpi^*)}\right), \\ \sigma\left(\varpi_{2\rho_k}; [\mathcal{Q}\varpi^*]_{\alpha_\Delta(\varpi^*)}\right), \sigma\left(\varpi^*; \left[\mathfrak{D}\varpi_{2\rho_k}\right]_{\alpha_\Delta\left(\varpi_{2\rho_k}\right)}\right)\end{matrix}\right\}$$

$$\leq \left[\Theta\left(\sigma\left(\varpi_{2\rho_k}, \varpi^*\right)\right)\right]^k + \Delta \min\left\{\begin{matrix}\sigma\left(\varpi_{2\rho_k}, \varpi_{2\rho_k+1}\right), \sigma\left(\varpi^*; [\mathcal{Q}\varpi^*]_{\alpha_\Delta(\varpi^*)}\right), \\ \sigma\left(\varpi_{2\rho_k}; [\mathcal{Q}\varpi^*]_{\alpha_\Delta(\varpi^*)}\right), \sigma\left(\varpi^*, \varpi_{2\rho_k+1}\right)\end{matrix}\right\}, \qquad (2.22)$$

as $\rho \to \infty$ using (2.22) and Θ_4, we get

$$1 < \Theta\left[\sigma\left(\varpi^*, [\mathcal{Q}\varpi^*]_{\alpha_\Delta(\varpi^*)}\right)\right] \leq 1.$$

Which is a contradiction. Hence $\varpi^* \in [\mathcal{Q}\varpi^*]_{\alpha_\Delta(\varpi^*)}$. Similarly, one can easily prove that $\varpi^* \in [\mathfrak{D}\varpi^*]_{\alpha_\Delta(\varpi^*)}$. Thus $\varpi^* \in [\mathfrak{D}\varpi^*]_{\alpha_\Delta(\varpi^*)} \cap [\mathcal{Q}\varpi^*]_{\alpha_\Delta(\varpi^*)}$.

The following result is a direct consequence of above Theorem by taking $\Delta = 0$.

Corollary 2.2: Let $(\Xi, \perp, \sigma)$ be an OCMS and $\{\mathfrak{D}, \mathcal{Q}\}$ are OLF-mapping from Ξ in to $\Im_\Delta(\Xi)$ orthogonal preserving and orthogonal continuous from Ξ in to $\Im_\Delta(\Xi)$ and for each $\alpha_\Delta \in \Delta\backslash\{0_\Delta\}$, $[\mathfrak{D}\varpi]_{\alpha_\Delta(\varpi)}, [\mathcal{Q}v]_{\alpha_\Delta(v)}$ are nonempty closed BS of Ξ. If $\exists\ \Theta \in F$ and $k \in (0,1)$, such that

$$\Pi\left([\mathfrak{D}\varpi]_{\alpha_L(\varpi)}, [\mathcal{Q}v\,]_{\alpha_\Delta(v)}\right) > 0 \implies \Theta\left(\Pi\left([\mathfrak{D}\varpi]_{\alpha_L(\varpi)}, [\mathcal{Q}v\,]_{\alpha_\Delta(v)}\right)\right) \leq \Theta\left(\sigma(\varpi, v)\right)^k$$

for all $\varpi, v \in \Xi$ with $\varpi \perp v$. Then $\mathfrak{D}$ and $\mathcal{Q}$ have a common orthogonal L-fuzzy fp.

Corollary 2.3: Suppose $(\Xi, \perp, \sigma)$ be an OCMS and let $\mathfrak{D}$ be an OLF-mapping from Ξ into $\Im_\Delta(\Xi)$, orthogonal preserving and orthogonal continuous and for each $\alpha_\Delta \in \Delta\backslash\{0_\Delta\}, [\mathfrak{D}\varpi]_{\alpha_\Delta(\varpi)}, [\mathfrak{D}v]_{\alpha_\Delta(v)}$ are nonempty closed BS of Ξ. If $\exists\ \Theta \in F$ and $k \in (0,1)$ and $\Delta \geq 0$, such that

$$\Theta\left[\Pi\left([\mathfrak{D}\varpi]_{\alpha_L(\varpi)}, [\mathfrak{D}v\,]_{\alpha_\Delta(v)}\right)\right] \leq \Theta\left(\sigma(\varpi, v)\right)^k + \Delta\, m(\varpi, v).$$

Where

$$m(\varpi, v) = \min\left\{\sigma\left(\varpi, [\mathfrak{D}\varpi]_{\alpha_L(\varpi)}\right), \sigma\left(v, [\mathfrak{D}v\,]_{\alpha_\Delta(v)}\right), \sigma\left(v, [\mathfrak{D}\varpi]_{\alpha_L(\varpi)}\right), \sigma\left(\varpi, [\mathfrak{D}v\,]_{\alpha_\Delta(v)}\right)\right\},$$

for all $\varpi, v \in \Xi$with $\varpi \perp v$ and $\Pi\left([\mathfrak{D}\varpi]_{\alpha_L(\varpi)}, [\mathfrak{D}v\,]_{\alpha_\Delta(v)}\right) > 0$. Then $\mathfrak{D}$ has an orthogonal L-fuzzy fp.

Corollary 2.4: Let $(\Xi, \perp\ \sigma)$ be an OCMS and let $\mathfrak{D}$ be an OLF-mapping, orthogonal preserving and orthogonal continuous from Ξ into $\Im_\Delta(\Xi)$ and for each $\alpha_\Delta \in \Delta\backslash\{0_\Delta\}, [\mathfrak{D}\varpi]_{\alpha_\Delta(\varpi)}, [\mathfrak{D}v]_{\alpha_\Delta(v)}$ are nonempty closed BS of Ξ. If $\exists\ \Theta \in F$ and $k \in (0,1)$ and $\Delta \geq 0$, such that

$$\Theta\left(\Pi\left([\mathfrak{D}\varpi]_{\alpha_L(\varpi)}, [\mathfrak{D}v\,]_{\alpha_\Delta(v)}\right)\right) \leq \Theta\left(\sigma(\varpi, v)\right)^k$$

for all $\varpi, v \in \Xi$ with $\varpi \perp v$ and $\Pi\left([\mathfrak{D}\varpi]_{\alpha_L(\varpi)}, [\mathfrak{D}v\,]_{\alpha_\Delta(v)}\right) > 0$. Then $\mathfrak{D}$ has an orthogonal L-fuzzy fp.

Theorem 2.5: Let $(\Xi, \perp, \sigma)$ be an OCMS and let $\{\mathfrak{D}, \mathcal{Q}\}$ be an OF-mapping from Ξ in to $\Im(\Xi)$, orthogonal preserving and orthogonal continuous and for each $\alpha \in (0,1], [\mathfrak{D}\varpi]_{\alpha(\varpi)}, [\mathcal{Q}v]_{\alpha(v)}$ are nonempty closed BS of Ξ. If $\exists\ \Theta \in F$ and $k \in (0,1)$ and $\Delta \geq 0$ such that

$$\Theta\left(\Pi\left([\mathfrak{D}\varpi]_{\alpha(\varpi)}, [\mathcal{Q}v\,]_{\alpha(v)}\right)\right) \leq \Theta\left(\sigma(\varpi, v)\right)^k + \Delta\, m(\varpi, v).$$

Where

$$m(\varpi, v) = \min\left\{\sigma\left(\varpi, [\mathfrak{D}\varpi]_{\alpha(\varpi)}\right), \sigma\left(v, [\mathcal{Q}v\,]_{\alpha(v)}\right), \sigma\left(v, [\mathfrak{D}\varpi]_{\alpha(\varpi)}\right), \sigma\left(\varpi, [\mathcal{Q}v\,]_{\alpha(v)}\right)\right\},$$

for all $\varpi, v \in \Xi$ with $\varpi \perp v$ and $\Pi\left([\mathfrak{D}\varpi]_{\alpha(\varpi)}, [\mathcal{Q}v\,]_{\alpha(v)}\right) > 0$ Then $\mathfrak{D}$ and $\mathcal{Q}$ have a common orthogonal fuzzy fp.

Proof: consider an OLF-mapping $\mathcal{J}: \Xi \to \Im_\Delta(\Xi)$ is orthogonal preserving then $\mathcal{J}\varpi \perp \mathcal{J}v$ or $\mathcal{J}v \perp \mathcal{J}\varpi$ for all $\varpi, v \in \Xi$ with $\varpi \perp v$ defined by

$$\mathcal{J}\varpi = X_{\Delta_{\mathfrak{D}(\varpi)}}.$$

Then for $\alpha_\Delta \in \Delta\backslash\{0_\Delta\}$,we have

$$[\mathcal{J}\varpi]_{\alpha_{L}(\varpi)} = \mathfrak{D}\varpi.$$

Hence by Theorem 2.1, we follow the result.

Taking $\Delta = 0$ in above result, we have following corollary.

Corollary 2.6: Let $(\Xi, \perp \ \sigma)$ be an OCMS and $\{\mathfrak{D}, \mathcal{Q}\}$ be an OF-mapping from Ξ in to $\Im(\Xi)$, orthogonal preserving and orthogonal continuous and for each $\alpha(\varpi) \in (0,1]$, $[\mathfrak{D}\varpi]_{\alpha(\varpi)}$ and $[\mathcal{Q}v]_{\alpha(v)}$ are nonempty closed BS of Ξ. If $\exists\ \Theta \in F$ and $k \in (0,1)$ such that

$$\Theta\left(\Pi\left([\mathfrak{D}\varpi]_{\alpha(\varpi)}, [\mathcal{Q}v\ ]_{\alpha(v)}\right)\right) \leq \Theta\left(\sigma(\varpi, v)\right)^{k}$$

for all $\varpi, v \in \Xi$ with $\varpi \perp v$ and $\Pi\left([\mathfrak{D}\varpi]_{\alpha(\varpi)}, [\mathcal{Q}v\ ]_{\alpha(v)}\right) > 0$. Then $\mathfrak{D}$ and $\mathcal{Q}$ have a common orthogonal fuzzy fp.

Example 2.7: Let $\Xi = [0,1]$, $\sigma(\varpi, v) = |\varpi - v|$, $\varpi, v \in \Xi$ with $\varpi \perp v \Leftrightarrow \varpi + v \geq 0$. Then $(\Xi, \perp, \sigma)$ is an OCMS. Let $\Delta = \{\eta, \omega, \tau, \kappa\}$ with $\eta \preccurlyeq_\Delta \omega \ \preccurlyeq_\Delta \kappa$ and $\eta \preccurlyeq_\Delta \tau \ \preccurlyeq_\Delta \kappa$, where ω and τ are not comparable, then $(\Delta, \preccurlyeq_\Delta)$ is an orthogonal complete distributive lattice. Define $\mathfrak{D}, \mathcal{Q}: \Xi \to \Im_\Delta(\Xi)$ as follows:

$$\mathfrak{D}(\varpi)(t) = \begin{cases} \kappa & \text{if } 0 \leq t \leq \frac{\varpi}{6} \\ \omega & \text{if } \frac{\varpi}{6} \leq t \leq \frac{\varpi}{3} \\ \tau & \text{if } \frac{\varpi}{3} \leq t \leq \frac{\varpi}{2} \\ \eta & \text{if } \frac{\varpi}{2} \leq t \leq 1 \end{cases}$$

$$\mathcal{Q}(\varpi)(t) = \begin{cases} \kappa & \text{if } 0 \leq t \leq \frac{\varpi}{12} \\ \eta & \text{if } \frac{\varpi}{12} \leq t \leq \frac{\varpi}{8} \\ \omega & \text{if } \frac{\varpi}{8} \leq t \leq \frac{\varpi}{4} \\ \tau & \text{if } \frac{\varpi}{4} \leq t \leq 1. \end{cases}$$

Let $\Theta(t) = e^{\sqrt{t}} \in F$ for $t > 0$ and for all $\varpi \in \Xi$, there exist $\alpha_\Delta(\varpi) = \kappa$, such that

$$[\mathfrak{D}\varpi]_{\alpha_\Delta(\varpi)} = \left[0, \frac{\varpi}{6}\right],$$

$$[\mathcal{Q}\varpi]_{\alpha_\Delta(\varpi)} = \left[0, \frac{\varpi}{12}\right].$$

Additionally, 0 is a common fp of $\mathfrak{D}$ and Q and all criteria of Theorem 2.1 are met.

3. Applications to domain of words

Assume Ω is a nonempty alphabet and Ω^∞ is the collection of all finite and infinite orthogonal sequences over Ω, with the understanding that the empty sequence ϕ is an element of Ω^∞. Additionally, on Ω^∞ we take into account the prefix order $\Leftrightarrow$ supplied by:

$$\varpi \Leftrightarrow v \quad \Leftrightarrow \varpi \text{ is a prefix of } v,$$

for each $\varpi \neq \phi \in \Omega^\infty$ denote by $l(\varpi)$ the length of ϖ. Then $l(\varpi) \in [0, \infty]$, whenever $\varpi \neq \phi$ and $l(\phi) = 0$. For each $\varpi, v \in \Omega^\infty$, let $\varpi \sqcap v$ be the common prefix of ϖ and v. Clearly, $\varpi = v$ if and only if

$$\varpi \Leftrightarrow v \text{ and } v \Leftrightarrow \varpi,$$

and $l(\varpi) = l(v)$. Then, the orthogonal Baire metric $\sigma_{\Leftrightarrow}$ is defined on $\Omega^\infty \times \Omega^\infty$ by

$$\begin{cases} \sigma_{\Leftrightarrow}(\varpi, v) = 0 & \text{if } \varpi = v \\ \sigma_{\Leftrightarrow}(\varpi, v) = 2^{-l(\varpi \sqcap v)} & \text{otherwise,} \end{cases}$$

such that the $\varpi \perp v \Leftrightarrow \varpi + v \geq 0$ then the OMS $(\Omega^\infty, \perp, \sigma_{\Leftrightarrow})$ is an orthogonal complete. Certainly, we assign to the average case time complexity analysis of the Quicksort divide- and- conquer sorting algorithm in [24]. Exactly, we deal with the following recurrence relation, $\mathfrak{R}(1) = 0$ and

$$\mathfrak{R}(\rho) = \frac{2(\rho - 1)}{\rho} + \frac{\rho + 1}{\rho}\mathfrak{R}(\rho - 1), \quad \rho \geq 2. \qquad (3.1)$$

Consider $\Omega = \mathbb{R}^+$. We accomplice to $\mathfrak{R}$ the functional $\Phi: \Omega^\infty \to \Omega^\infty$ given by

$$\left(\Phi(\varpi)\right)_1 = \mathfrak{R}(1)$$

and

$$\left(\Phi(\varpi)\right)_\rho = \frac{2(\rho - 1)}{\rho} + \frac{\rho + 1}{\rho}\varpi_{\rho-1},$$

$\forall \rho \geq 2$ (if $\varpi \in \Omega^\infty$ has length $\rho < \infty$, we write $\varpi \coloneqq \varpi_1\varpi_2 \ldots \varpi_\rho$ and if ϖ is an infinite word we write $\varpi \coloneqq \varpi_1\varpi_2 \ldots$). It follows by the construction that $l\left(\Phi(\varpi)\right) = l(\varpi) + 1$ for all $\varpi \in \Omega^\infty$ and $l\left(\Phi(\varpi)\right) = +\infty$ whenever, $l(\varpi) = +\infty$. Let $\mathfrak{D}: \Omega^\infty \to \mathfrak{J}(\Omega^\infty)$ be the OLF-mapping given by

$$\mathfrak{D}_\varpi = \left(\Phi(\varpi)\right)_{\alpha_\Delta} \text{ for all } \varpi \in \Omega^\infty \text{ with } (\varpi \perp \mathfrak{D}_\varpi) \text{or } (\mathfrak{D}_\varpi \perp \varpi) \text{ and } \alpha_\Delta \in \Delta \backslash \{0_\Delta\}.$$

and analyze the following two cases:

Cases1: If $\varpi \perp v$ and $\varpi = v$, then, we have

$$\Pi_{\hat{\mathfrak{c}}}\left(\left(\Phi(\varpi)\right)_{\alpha_\Delta},\left(\Phi(\varpi)\right)_{\alpha_\Delta}\right) = 0 = \sigma_{\hat{\mathfrak{c}}}(\varpi,\varpi).$$

Case2: If $\varpi \perp v$ and $\varpi \neq v$ then, we write

$$\Pi_{\hat{\mathfrak{c}}}\left(\left(\Phi(\varpi)\right)_{\alpha_\Delta},\left(\Phi(v)\right)_{\alpha_\Delta}\right) = \sigma_{\hat{\mathfrak{c}}}\left(\left(\Phi(\varpi)\right)_{\alpha_\Delta},\left(\Phi(v)\right)_{\alpha_\Delta}\right)$$

$$= 2^{-\left(l\left(\Phi(\varpi)\right)_{\alpha_\Delta}\sqcap\left(\Phi(v)\right)_{\alpha_\Delta}\right)} = 2^{-\left(l\left(\Phi(\varpi\sqcap v)\right)_{\alpha_\Delta}\right)} = 2^{-(l(\varpi\sqcap v)+1)}$$

$$= \frac{1}{2}2^{-l(\varpi\sqcap v)} = \left(\frac{1}{\sqrt{2}}\right)^2 \sigma_{\hat{\mathfrak{c}}}(\varpi, v).$$

It is immediate to prove that all the circumstances of corollary 2.4 are verified with $\Theta(t) = e^{\sqrt{t}}$ and $k = \frac{1}{\sqrt{2}}$. Additionally, the OLF-mapping $\mathfrak{D}$ has an orthogonal L-fuzzy fp $\varpi = \varpi_1\varpi_2 \ldots \in \Omega^\infty$ that is $\varpi \in (\mathfrak{D}_\varpi)_{\alpha_\Delta}$. Also, in the light of the definition of $\mathfrak{D}, \varpi$ is a fp of Φ, and hence, ϖ solves the recurrence relation (3.1). We have $\varpi = 0$,

$$\varpi_\rho = \frac{2(\rho-1)}{\rho} + \frac{\rho+1}{\rho}\varpi_{\rho-1},\quad \rho \geq 2.$$

4. Conclusions

We proved some common orthogonal L-fuzzy fp results for almost orthogonal Θ-contraction in the setting of orthogonal complete ms by using the notion of orthogonal L-fuzzy mappings. We also presented an application to domain of words which shows the significance of the investigation of this paper.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References

[1] Banach, S. Sur les op´erations dans les ensembles abstraits et leur application aux ´equations int´egrales, *Fundamenta Mathematicae*, **3**, 133–181 (1922).

[2] Caccioppoli, R. Un teorema generale sull' esistenza di elementi uniti in una trasformazione funzionale, *Rendiconti Lincei*, **11**, 794–799 (1930).

[3] Zadeh, L.A. Fuzzy sets, *Information and Control*. **8**(3), 338–353 (1965).

[4] Arora, S.C. and Sharma, V. Fixed points for fuzzy mappings, *Fuzzy Sets and Syst*. **110**, 127-130 (2000).

[5] Goguen, J. A. L- Fuzzy sets, *Journal of Mathematical Analysis and Applications*. **18**, 145–174 (1967).

[6] Abdullahi, M. S. and Azam, A. L-fuzzy Fixed Point Theorems for L-fuzzy Mappings via βFL - admissible with Applications. *Journal of Uncertainty Analysis and Applications.* **5**(2), (2017).

[7] Adibi, H. Cho, Y.J. O'Regan, D. and Saadati, R. Common fixed point theorems in L-fuzzy ms, *Appl. Math. Comput.* **182**, 820–828 (2006).

[8] A. Ahmad, A. Al-Rawashdeh, A. Azam, Fp results for {α, ξ}-expansive locally contractive mappings, *Journal of Inequalities and Applications*. **2014**(364), (2014).

[9] J. Ahmad, A. Azam, S. Romaguera, On locally contractive Fuzzy set-valued mappings, Journal of Inequalities and Applications, **2014**(74), (2014).

[10] J. Ahmad, N. Hussain, A. R. Khan, A. Azam. Fixed Point Results for Generalized Multi-valued Contractions. *The Journal of Nonlinear Sciences and Applications*. 909-918 (2015).

[11] J. Ahmad, A. E. Al-Mazrooei, Y. J. Cho, Y. O. Yang. Fixed point results for generalized Θ-contractions. *J. Nonlinear Sci. Appl.*, **10**, 2350–2358 (2017).

[12] A. Al-Rawashdeh and J. Ahmad, Common Fixed Point Theorems for JS- Contractions,Bulletin of *Mathematical Analysis and Applications*. **8**(4), 12-22 (2016).

[13] Gordji, M.E., Rameani, M., de la Sen M., and Cho, Y. On orthogonal sets and Banach fixed point theorem, *Fixed Point Theory*, **18**,569–578 (2017).

[14] Uddin, F., Park, C., Javed, K., Arshad, M., and Lee, J.R. *Orthogonal m-metric space and an application to solve integral equations*, Advances in Difference Equations, 2021.

[15] Uddin, F., Javed, K., Aydi, H., Ishtiaq, U., and Arshad, M. Control Fuzzy Metric spaces via Orthogonality with an Application, *Journal of Mathematics*. **2021**, Article ID 5551833, (2021).

[16] Ishtiaq, U., Javed, K., Uddin, F., Sen, M. D. L., Ahmed, K., & Ali, M. U. Fixed point results in orthogonal neutrosophic metric space. *Complexity*, **2021**, (2021).

[17] Farheen, M., Ahmed, K., Javed, K., Parvaneh, V., Din, F. U., & Ishtiaq, U. Intuitionistic Fuzzy Double Controlled Metric Spaces and Related Results. *Security and Communication Networks*, **2022**, (2022).

[18] Ahmada, J., Mazrooeia, A.E.A., Rassiasb,T.M., Common Fixed Point Theorems with Applications to Theoretical Computer Science, *Int. J. Nonlinear Anal. Appl*. 2008-6822.

[19] Rashid, M. Azam, A. and Mehmood, N. L-fuzzy fixed points theorems for L-fuzzy mappings via βFL -admissible pair**. Sci. World J**. **2014**, 1–8 (2014).

[20] Jleli, M. and Samet, B. A new generalization of the Banach contraction principle. *J. Inequal. Appl.* **2014**, 38 (2014).

[21] Hancer, HA. Minak, G. and Altun, I. On a broad category of multivalued weakly Picard operators. *Fixed Point Theory*, **18**(1), 229-236, (2017).

[22] N. Hussain, J. Ahmad and A. Azam. On Suzuki-Wardowski type fixed point theorems, *J. Nonlinear Sci. Appl*. **8**, 1095-1111 (2015).

[23] Azam, A. Arshad, M. and Vetro, P. On a pair of fuzzy ϕ-contractive mappings. *Math. Comput. Model*. **52**, 207-214 (2010).

[24] Saadati, R., Vaezpour, S. M., & Cho, Y. J. Quicksort algorithm: application of a fixed point theorem in intuitionistic fuzzy quasi-metric space at a domain of words. *Journal of Computational and Applied Mathematics*, **228**(1), 219-225 (2009).